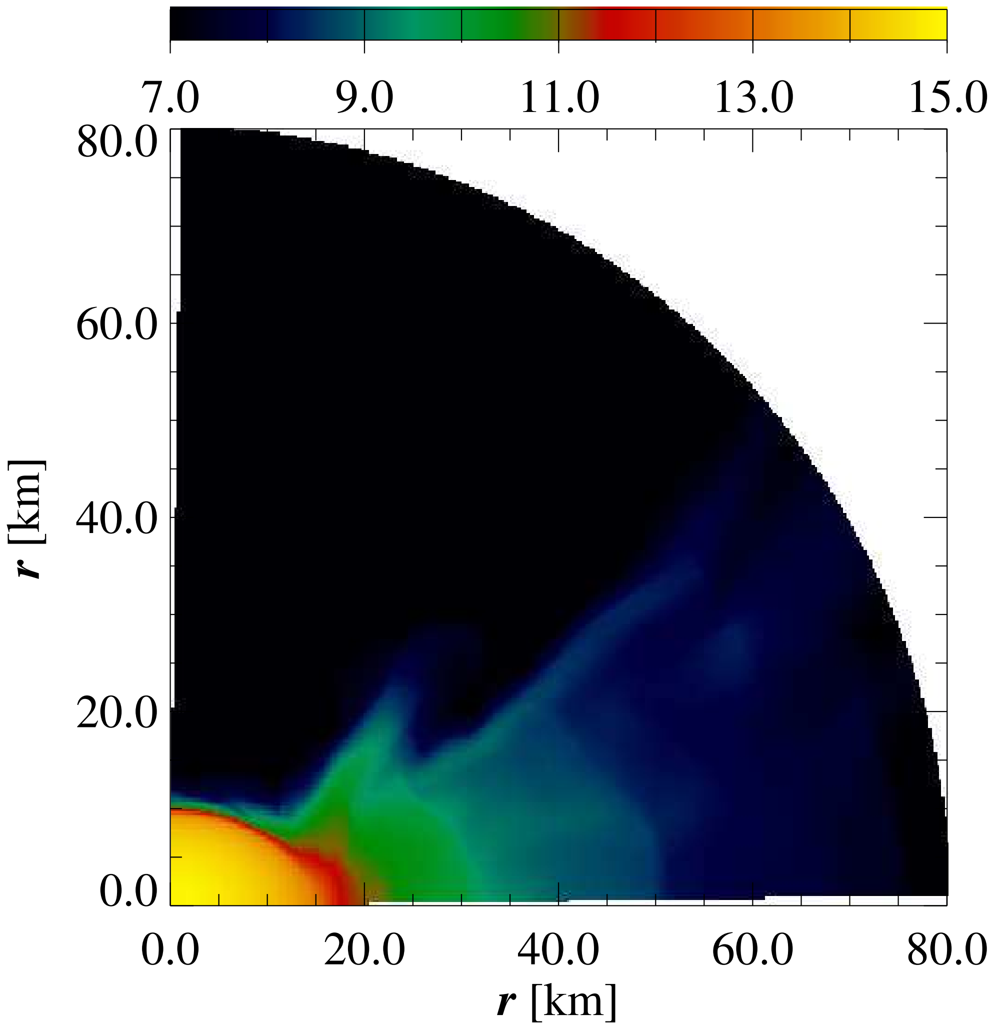


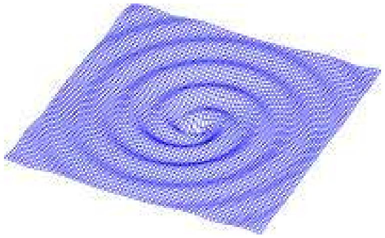
## Harald Dimmelmeier

# Nonlinear Effects in Pulsations of Rotating Neutron Stars

Presented work in collaboration with

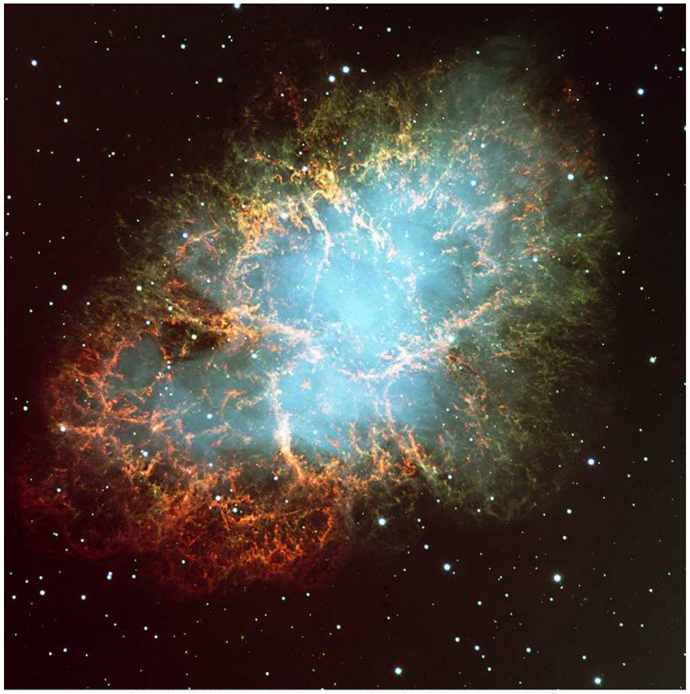
Nick Stergioulas (Aristotle University Thessaloniki)
Toni Font (Universidad de Valencia)

Dimmelmeier, Stergioulas, and Font,
*Mon. Not. R. Astron. Soc.*, 2006, submitted
(`astro_ph/0511394`)



DFG SFB Transregio 7 “Gravitational Wave Astronomy”

# Pulsations of Rotating Neutron Stars



The Crab Nebula in Taurus (VLT KUEYEN + FORS2)

ESO PR Photo 40f/99 ( 17 November 1999 )

© European Southern Observatory

Excitation mechanisms for pulsations:

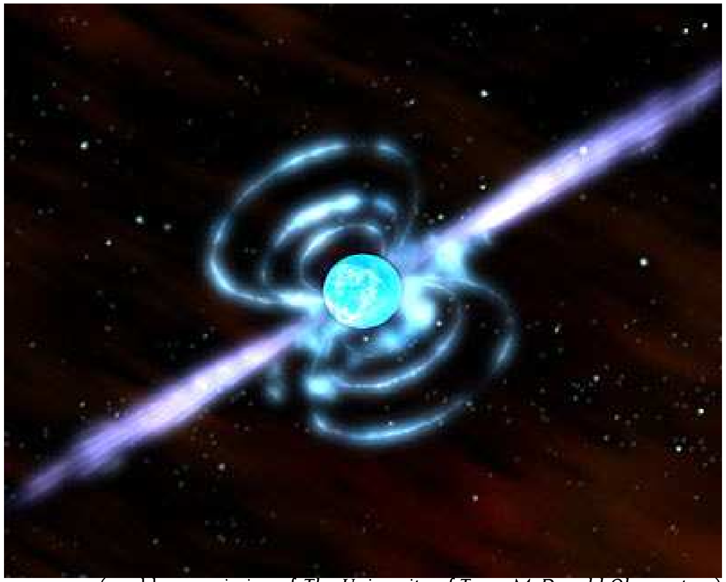
- Rotational supernova core collapse.
- Accretion-induced collapse.
- Core quakes due to strong phase transitions in EoS.
- Formation of hypermassive neutron star
  (from binary neutron star merger).

Potential source of detectable HF gravitational waves!

Aim: Use wave signal to determine neutron star structure
and constrain high density EoS.

For nonrotating neutron star models:
Theory of asteroseismology already exists
(e.g. Andersson and Kokkotas, 1998;
Benhar, Ferrari, Gualtieri, 2004).

## Previous Work


(used by permission of *The University of Texas McDonald Observatory*)

In spherical symmetry or slow rotation limit:
Use perturbative methods to determine mode frequencies.

Possible since few years:
Study pulsations of rotating neutron stars
with fully nonlinear evolution codes.

- Several studies in Cowling approximation
  (e.g. Font et al., 2001; Yoshida et al., 2002;
  Stergioulas, Apostolatos, Font, 2004)

  Result: Frequencies in Cowling wrong by up to 100%!

  Hope: Use Cowling simulations to establish empirical relation for correct mode frequencies.

- Full evolution using Cactus with GR-Hydro (Font et al., 2002).

  Very limited set of models and modes
  (computationally expensive 3d code, no high resolution)

Review article: Nick Stergioulas, “Rotating Stars in Relativity”,
*Living Rev. Relativity*, 3, 2003, `http://www.livingreviews.org/lrr-2003-3`.

## New Nonlinear Simulations

In nonlinear codes:
Excite pulsations with small amplitude perturbations, use FFT to extract mode frequencies.

General advantages of nonlinear codes:

- Can also study nonlinear phenomena (mass-shedding-induced damping, instabilities, mode coupling, avoided crossing).
- Can obtain information about relative mode strengths.


(www.madlantern.com)

We use CoCoNuT code:

- Full spacetime evolution (no Cowling approximation).
- Conformal flatness approximation for $3+1$ metric equations (CFC – excellent and tested approximation for rotating neutron stars).
- HRSC methods for hydrodynamic equations and spectral methods for metric equations (Mariage des Maillages).
- Spherical polar coordinates, equidistant grid inside star ($160 \times 60$ grid points).
- Axisymmetry and equatorial symmetry (computationally fast).

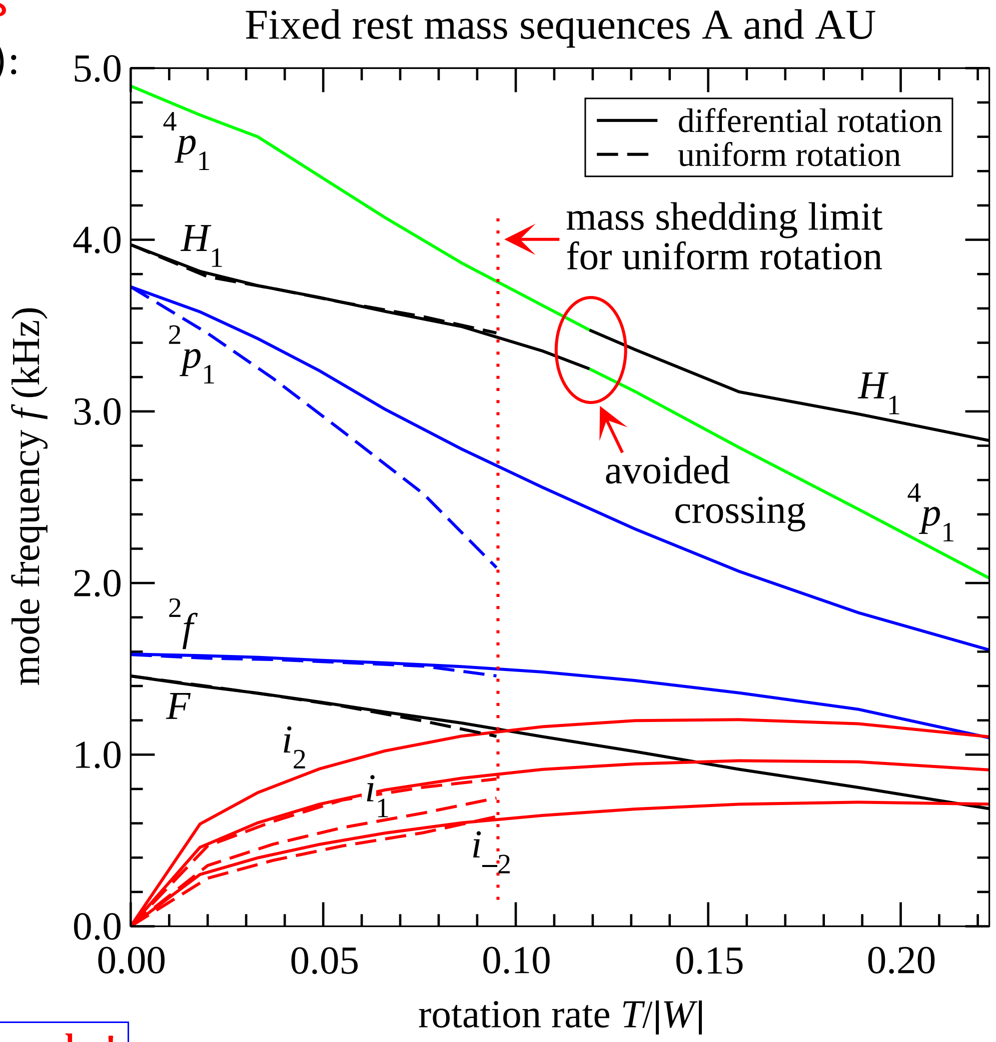
# Models and Linear Mode Frequencies

We simulated 4 sequences of equilibrium models (same as in Stergioulas, Apostolatos, Font, 2004):

- A: Fixed rest mass, differential rotation.
- AU: Fixed rest mass, uniform rotation.
- B: Fixed central density, differential rotation.
- BU: Fixed central density, uniform rotation.

Example:

Dependence of mode frequencies on rotation ($\boldsymbol{F}$ and $\boldsymbol{H}_1$-mode, ${}^2\boldsymbol{f}$ and ${}^2\boldsymbol{p}_1$-mode, ${}^4\boldsymbol{p}_1$-mode).



Obvious avoided crossing between $\boldsymbol{H}_1$ and ${}^4\boldsymbol{p}_1$-mode!

Observe also low-frequency inertial modes.

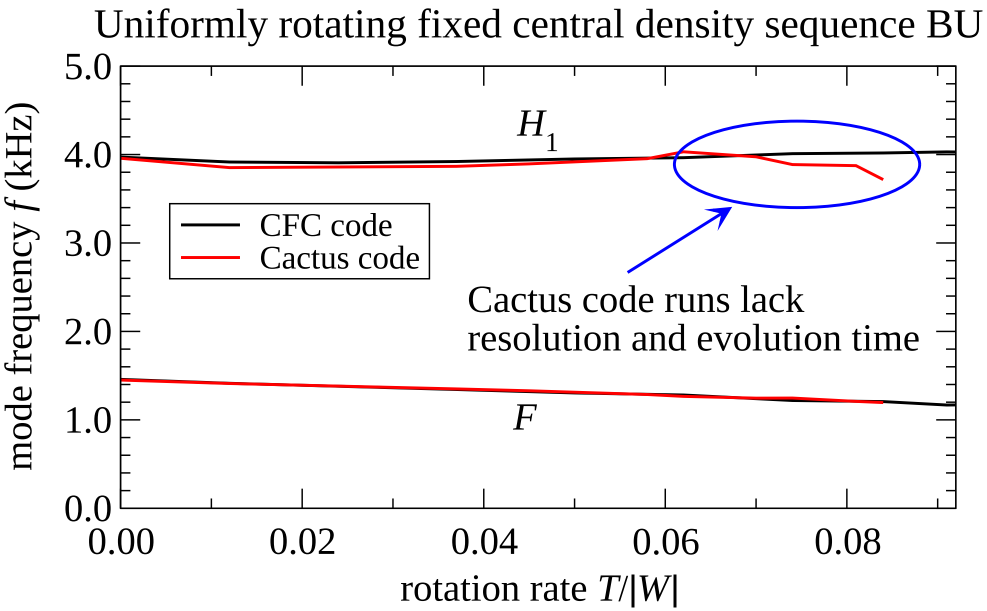
# Comparison to Fully General Relativistic Results

Sequence BU was already simulated with Cactus GR-Hydro code (Font et al., 2002).

Comparison with our results:

- Excellent agreement for fundamental $\boldsymbol{F}$-mode.
- Good agreement for first overtone $\boldsymbol{H}_1$.

  Close to mass-shedding limit:
  Cactus code runs probably have insufficient resolution or boundary problems.



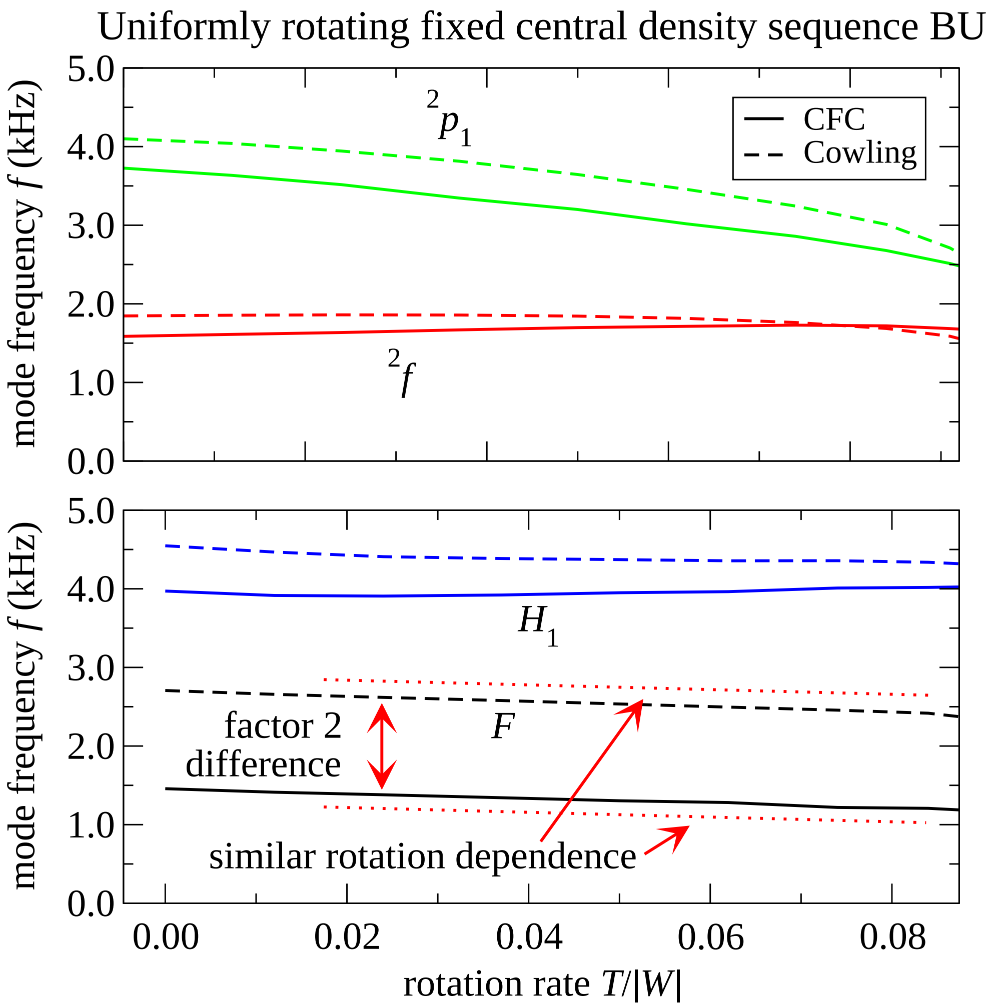
# Comparison to Results in Cowling Approximation

For $\boldsymbol{F}$ and $^2\boldsymbol{p}_1$-mode:
Cowling simulation reproduce correct rotation dependence reasonably well.

But: $\boldsymbol{F}$-mode frequencies too high
by factor $\sim 2$ in Cowling ($\sim 1$ kHz).

For $\boldsymbol{H}_1$ and $^2\boldsymbol{f}$-mode:
Cowling simulations yield frequencies similar to correct ones.

But: Rotation trend not clearly reproduced
(dependence is small anyway).



Idea in Font et al., 2001; Stergioulas, Apostolatos, Font, 2004:

Use results from Cowling simulations and empirical relation to obtain correct frequencies!

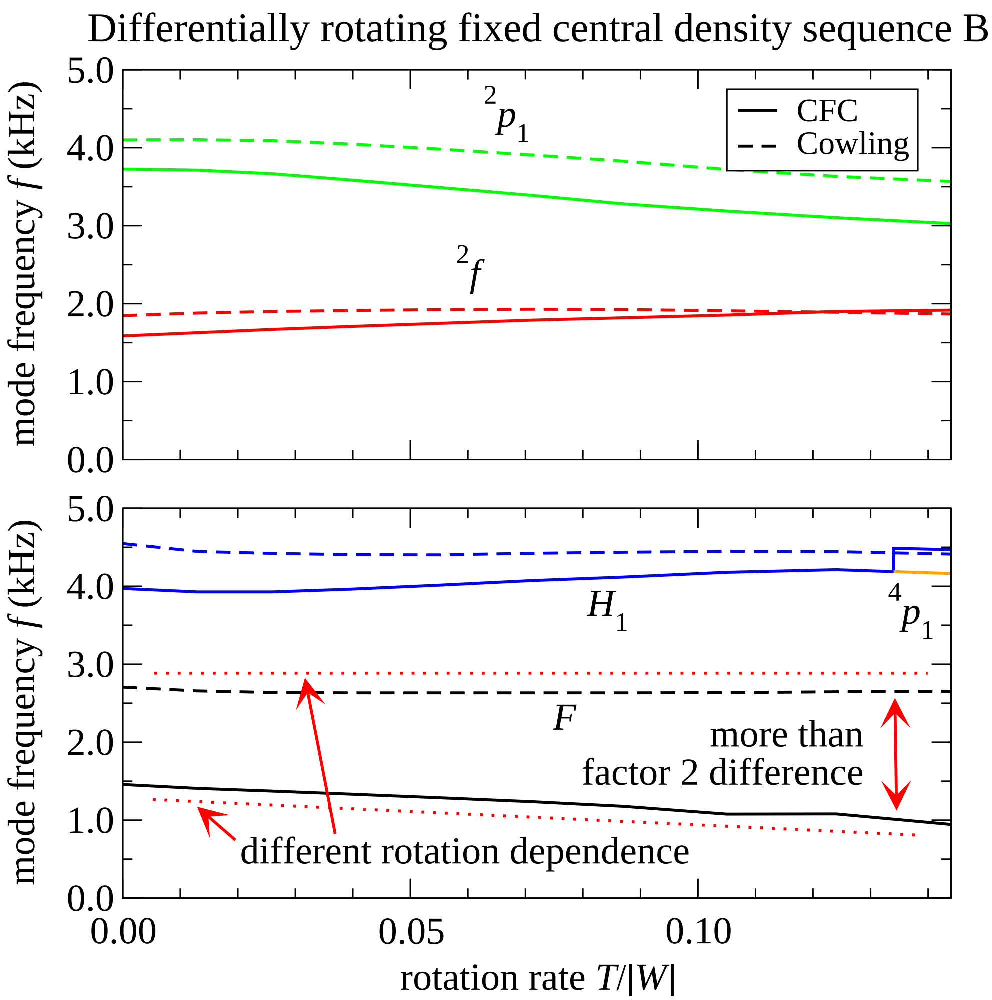
# Failure of Empirical Relations

We have checked these empirical relations for

- $F$-mode frequency in sequence BU
  (Font et al., 2001)

  Relation yields accuracy of better than 2%!

- $F$ and ${}^{2}f$-mode frequency in sequence A
  (Stergioulas, Apostolatos, Font, 2004;
  using information about compactness of
  models from Yoshida and Eriguchi, 2001).



Predicted uncertainty of these relations:
Few percent.

For most rapidly rotating model of sequence A:

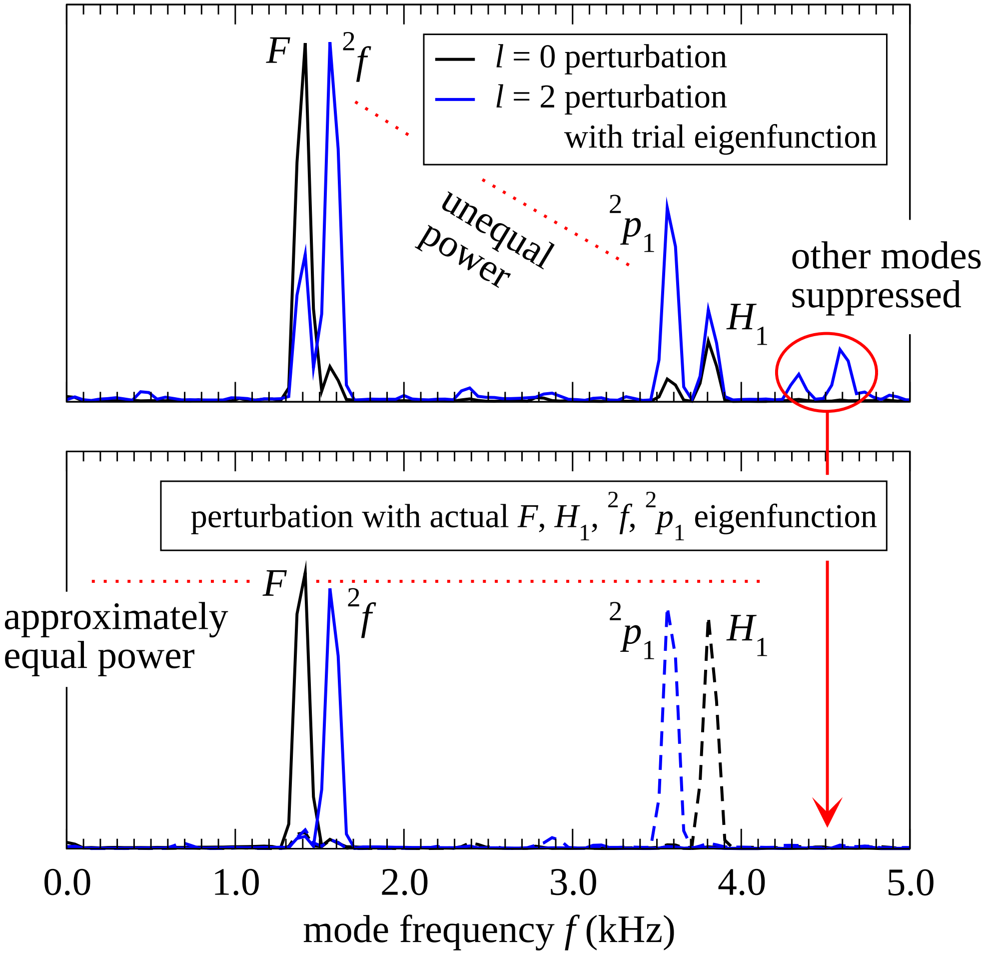
Difference of $\sim 30\%$ for $F$ and ${}^{2}f$-mode!

Bottomline: Such relations must be used with caution!

# Eigenfunction Recycling

As initial perturbation to excite pulsations:
Use analytic “trial” eigenfunctions
(for various parity, $l = 0, 2, 4$).

These differ from exact eigenfunction.
$\Longrightarrow$ They also excite unwanted modes
(e.g. higher order harmonics).



New approach: Use eigenfunction “recycling”!

- Extract eigenfunction of selected mode (at specific frequency).
- Use extracted eigenfunction as initial perturbation in second run (“recycling run”).
- Convenient check:
  Compare eigenfunctions from original run and recycling run.

Can select single mode for excitation and efficiently suppress all other modes!

With appropriate choice of perturbation amplitude: Get constant peak height in PSD.

# Nonlinear Harmonics

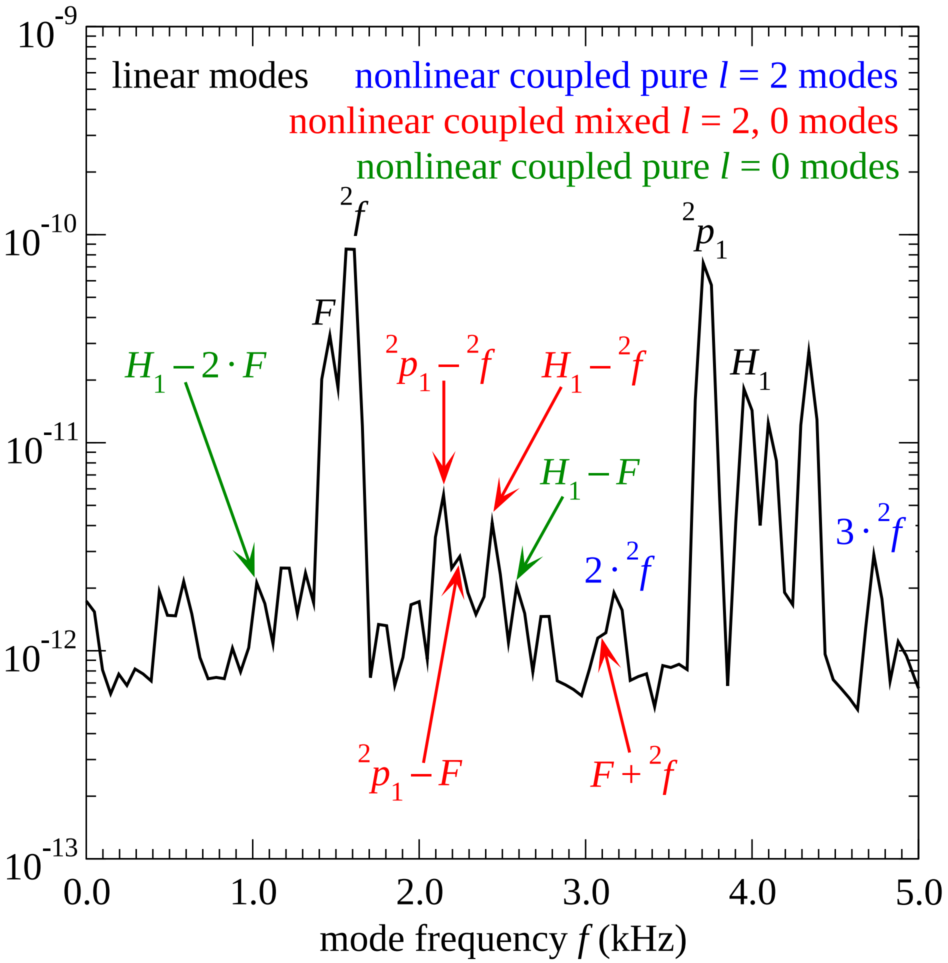
Genuine nonlinear effect:

Nonlinear harmonics of linear pulsation modes
(sums and differences of linear modes,
including self-couplings).

Recently: Such nonlinear harmonics observed in oscillating tori around Kerr black holes (Zanotti et al., 2005).

Example: Nonrotating star,
excited by $l = 2$ trial eigenfunction.

Observe also coupling of $l = 0$ and $l = 2$ modes!



In linear approximation for nonrotating star: Modes of different $l$ are orthogonal to each other.

Two effects:
- Approximate nature of $l = 2$ eigenfunction: $l = 0$ modes are also excited.
- Nonlinear effects couple all linear modes even with different $l$.

Criterion to distinguish linear modes and harmonics: Scaling with perturbation amplitude.

# Nonlinear 3-Mode Couplings

Rotation effects modes differently.

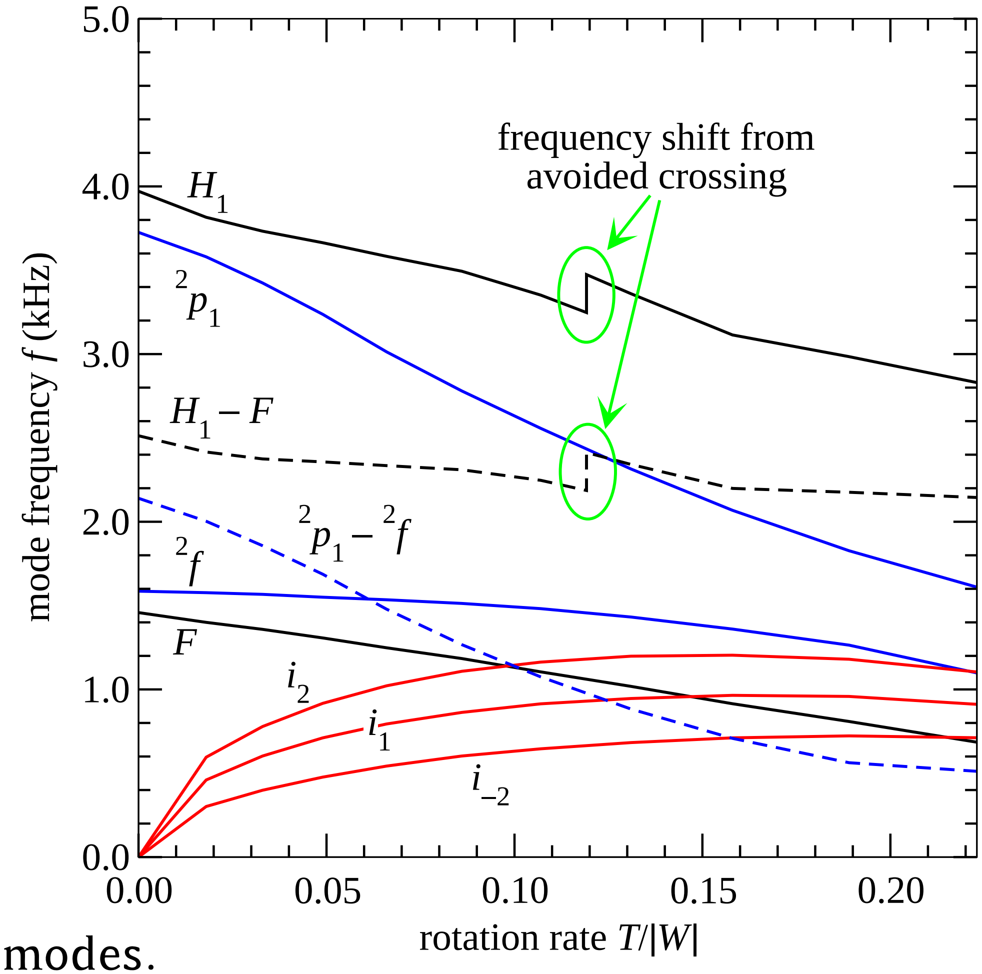
$\Longrightarrow$ When neutron star is rotating:
Presence of nonlinear harmonics opens
possibility for 3-mode couplings.

Modes interact when frequencies cross!

Example: $\boldsymbol{H_1 - F}$-mode and ${}^2\boldsymbol{p}_1$-mode.

Other possibility:
Mode crossing with some inertial mode.

This could potentially lead to resonance effects
or even parametric instabilities.
$\Longrightarrow$ Possibly significant energy transfer between modes.



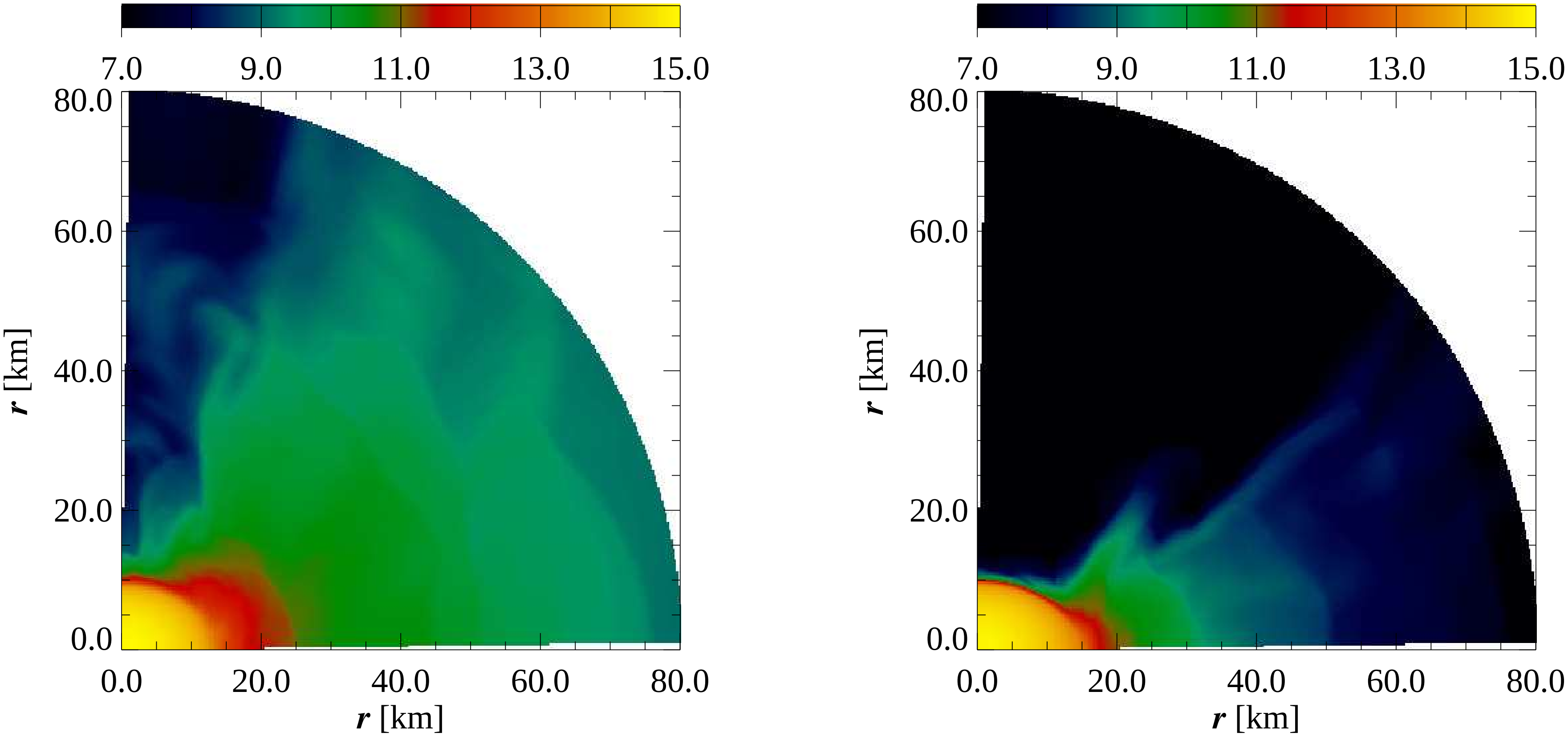
Most interesting case of energy transfer for gravitational waves:
Strong oscillating, weakly radiating mode $\longrightarrow$ weakly oscillating, strongly radiating mode
(suggested by Clark, 1979; Eardly, 1983).

We only observe if necessary conditions are fulfilled; we perform no further studies.

# Mass-Shedding for Rapidly Rotating Neutron Stars

For small but finite pulsation amplitudes:
Mass-shedding occurs in (almost) maximally rotating neutron stars.

- In Cowling approximation (left): Mass-shedding creates extended high-entropy envelope.
- With coupled spacetime evolution (right): Mass-shedding strongly suppressed.



We do not know exact reason for this observed mechanism!

# Mass-Shedding-Induced Damping

Mass-shedding-induced damping is another striking nonlinear effect.

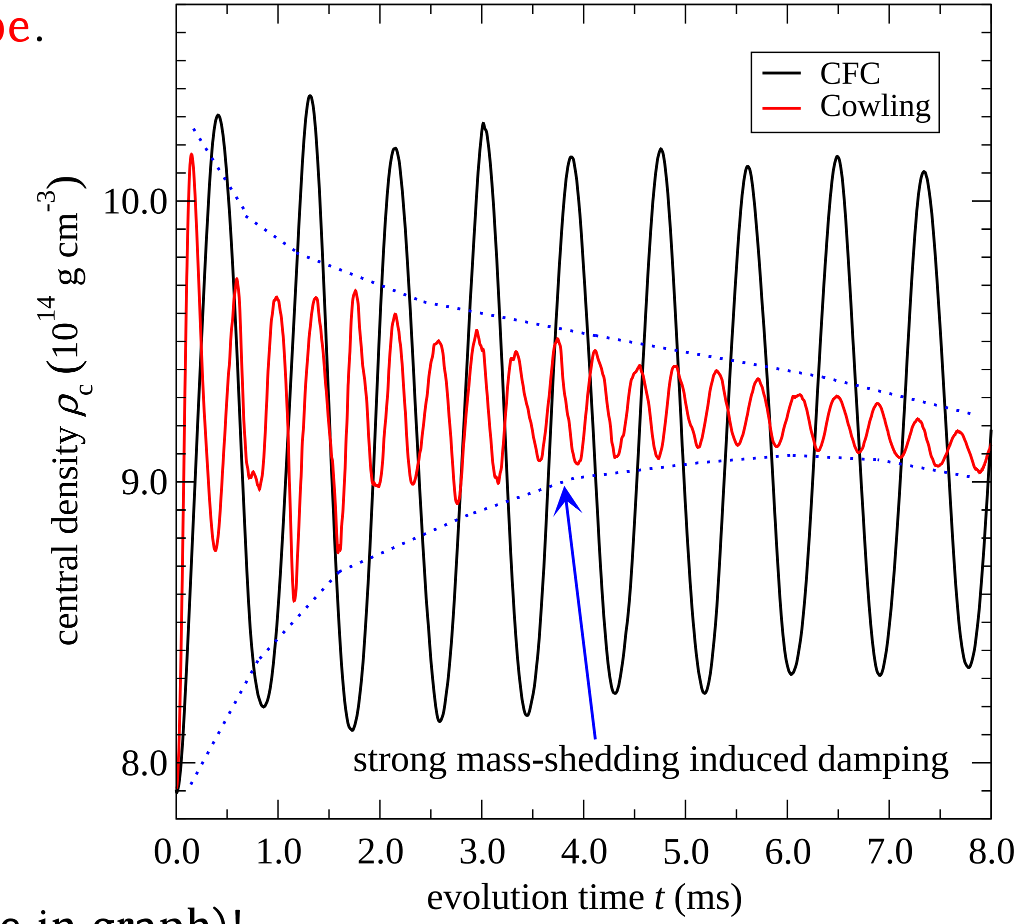
At each pulsation, star ejects mass into envelope.
⇒ Pulsation energy is lost, pulsation damped.

If damping time scale is comparable to growth time scale of unstable modes (e.g. CFS-instability driven $f$ or $r$-modes):

Damping mechanism could limit mode growth.

Example:

$l = m = 2$ $f$-mode becomes unstable only near mass-shedding limit.



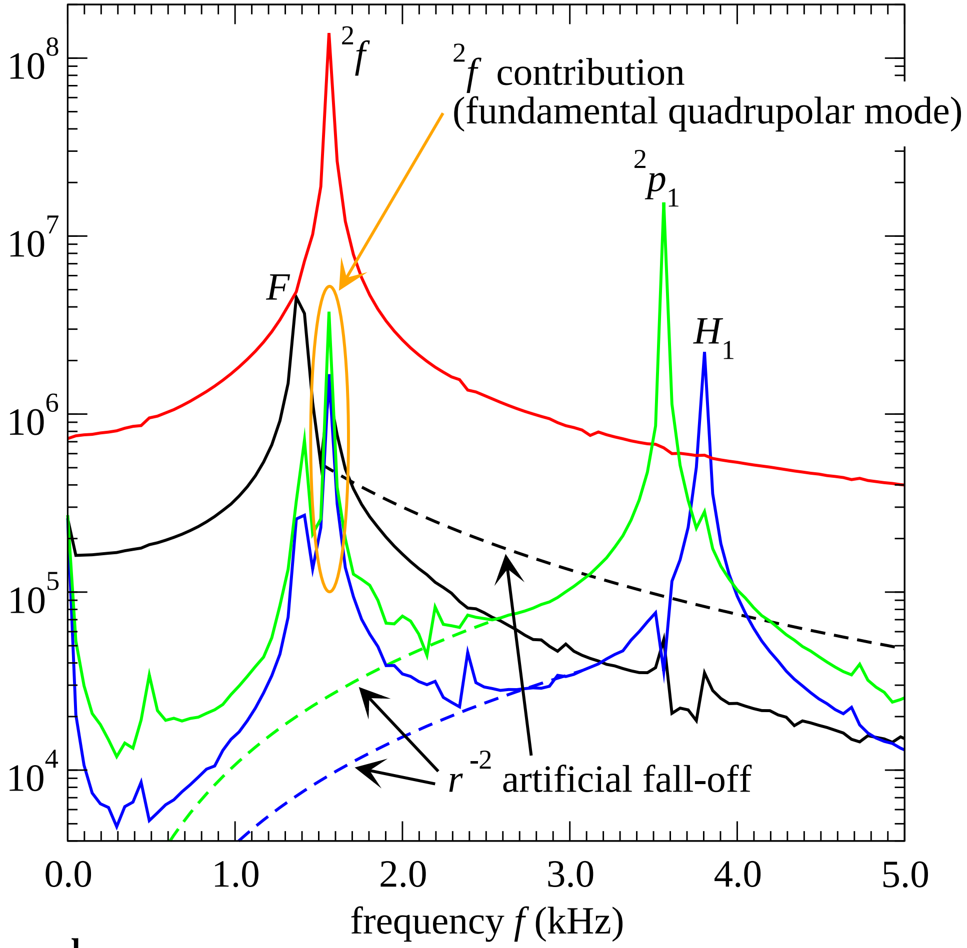
But with coupled spacetime evolution:
Much longer damping time scale (not detectable in graph)!

This could efficiently reduce this damping effect for unstable modes!

## Gravitational Wave Power Spectrum

We also extract gravitational waves emitted by pulsations.

We use Newtonian quadrupole formula
(in time-integrated form).



Example:
Gravitational wave power spectrum
for slowly rotating model from sequence A
(using eigenfunction recycling technique).

As expected:
Quadrupolar ${}^2f$-mode is strong emitter.

Observation:
Small leaking of energy into ${}^2f$-mode from other modes:
Creates strong contribution in signal.

$\Longrightarrow$ Suppress that contribution in signal processing.

# Detectability of Gravitational Waves

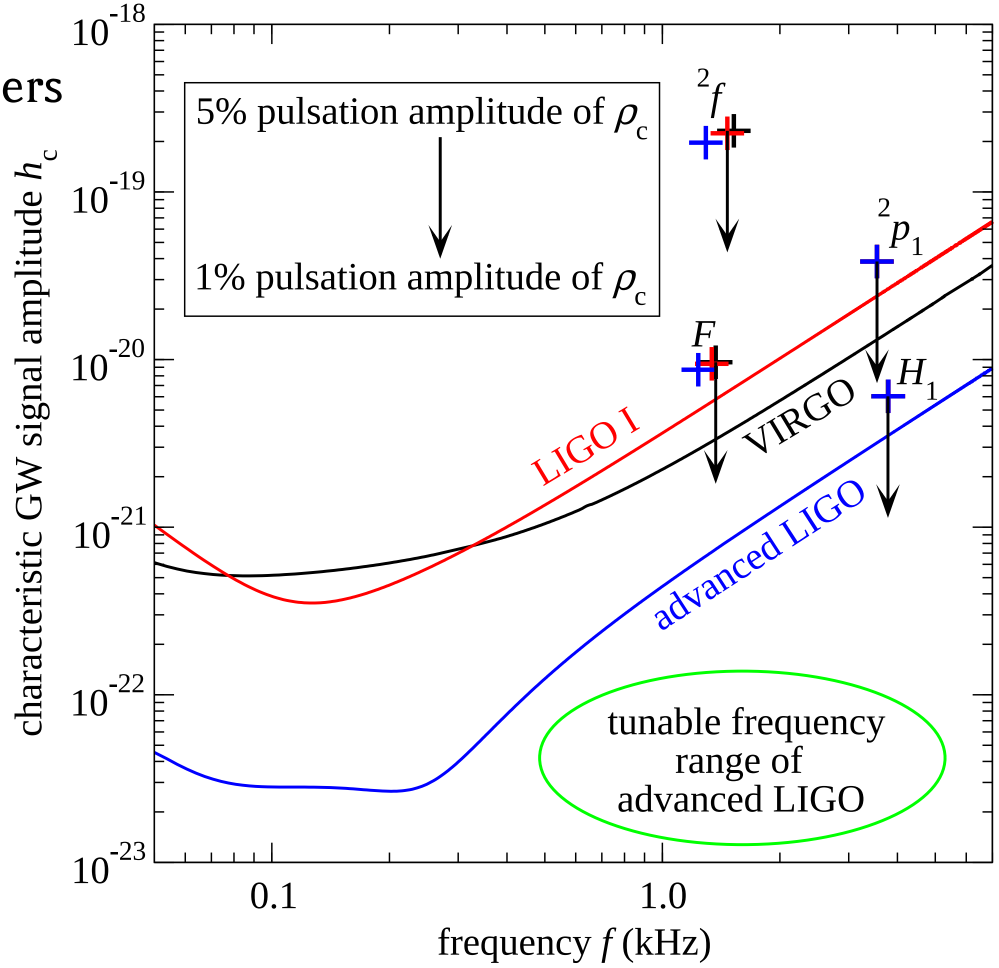
Estimate detectability prospects by interferometers (VIRGO, LIGO I, advanced LIGO):

Compute (slightly detector dependent)

- characteristic signal amplitude,
- characteristic signal frequency,
- signal-to-noise ratio.

Exploit linear scaling properties:

- Initial perturbation amplitude.
- Pulsation amplitude during evolution.
- Gravitational wave signal amplitude.
- Square root of signal duration time.



Can construct relation between detectability and required pulsation amplitude.

Location of signals along ascending high frequency slope of detector:
Dependence of mode frequency on rotation is crucial for detection.