Binary Black Holes Evolutions

with the BAM code

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Outline

Punctures.

Solving the constraints. (Brandt and Bruegmann 1997)

Writing the 3-metric and the extrinsic curvature as:

$$\begin{aligned} \gamma_{ij} &= \psi^4 \delta_{ij} \\ K_{ij} &= \psi^{-2} \tilde{K}_{ij} \end{aligned}$$

the constraints can be written as:

$$\begin{aligned} \nabla_i K^{ij} &= 0 \\ \triangle \psi + \frac{1}{8} K_{ij} K^{ij} \psi^{-7} &= 0 \end{aligned}$$

For a Schwarzschild black hole the constraints are solved by:

$$\Psi = 1 + \frac{m}{2r} \qquad K_{ij} = 0$$

For N black holes and non-vanishing extrinsic curvature we write:

$$\begin{aligned} \gamma_{ij} &= (\Psi_{BL} + u)^4 \delta_{ij} \\ \Psi_{BL} &= 1 + \sum_{i=1}^{n} \frac{m_i}{2r_i} \end{aligned}$$

and

$$\begin{aligned} K_{ij}^{PS} &= \frac{3}{2r^2}(P_i n_j + P_j n_i - (g_{ij} - n_i n_j) P_k n^k) \\ &+ \frac{3}{r^3}(\epsilon_{ikl} S^k n^l n_j + \epsilon_{jkl} S^k n^l n_i) \\ K_{ij} &= \sum_{k=1}^{N} K_{ij\,(k)}^{PS} \end{aligned}$$

The momentum contraint is solved exactly and the hamiltonian constraint becomes an elliptic equation for u.

Fixed vs moving punctures

- Fixed
  - Singular behavior contained in ψ_{BL} and treated analytically.
  - Shift is zero at the puncture.
  - Gauge.
    - Lapse
      $\partial_t \alpha = -2\alpha\psi^4 K$
    - Shift
      $\partial_t \beta^i = \frac{3}{4}\alpha\psi^{-2}B^i, \qquad \partial_t B^i = \partial_t \tilde{\Gamma}^i - \eta B^i$
- Moving
  - The conformal factor is evolved numerically:
    - Goddard 2005: ϕ where $\psi^4 = e^{4\phi}$
    - UTB 2005: $\chi = e^{-4\phi}$
  - Shift is NOT zero at the puncture.
  - Gauge.
    - Lapse
      $\partial_t \alpha = \beta^i \partial_i \alpha - 2\alpha K$
    - Shift
      $\partial_t \beta^i = \frac{3}{4}\alpha B^i, \qquad \partial_t B^i = \partial_t \tilde{\Gamma}^i - \beta^j \partial_j \tilde{\Gamma}^i - \eta B^i$

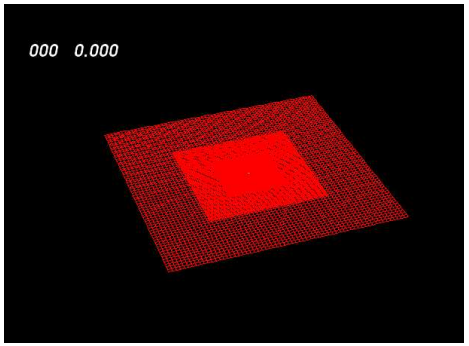
BAM code

- BSSN
- Moving punctures (ϕ and χ)
- Moving boxes
- Radiative boundary conditions
- Mixed second and fourth order in space and time:
  - 4th order stencils
  - evolution with RK4
  - 2nd order interpolation in time in the refinement boundaries.

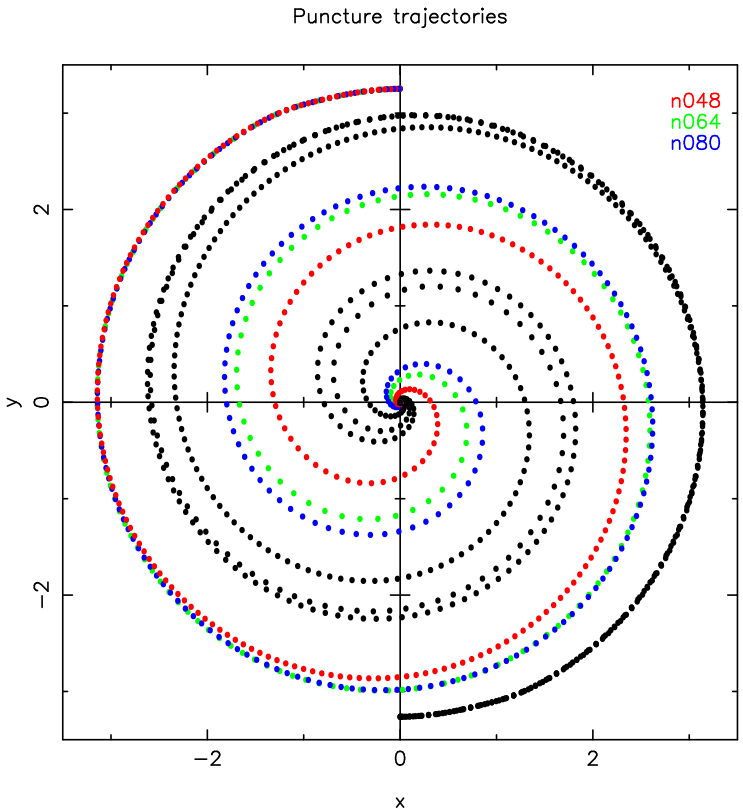
R=3.257

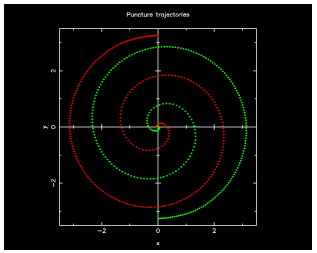
$$m = 0.483 \qquad P_{\pm} = 0.133 \qquad M_0 = 0.996$$

Number of points per level	$\{48^3, 64^3, 80^3\}$
Number of levels	$\{11, 11, 10\}$
Finest Resolutions	$\{\frac{2M}{64}, \frac{1.5M}{64}, \frac{1.2M}{64}\}$
Boundary	$\{768, 768, 384\}$

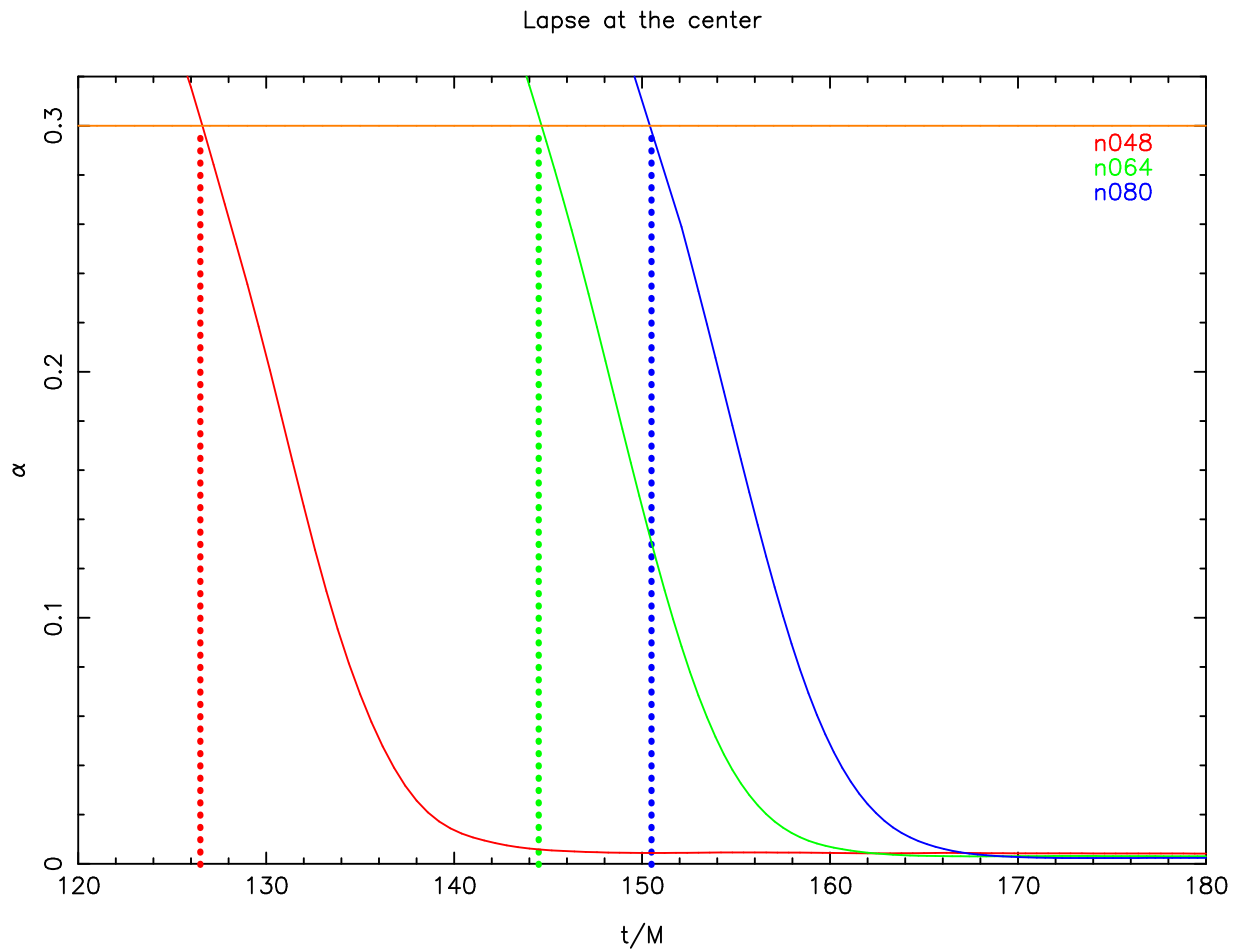


Trajectories



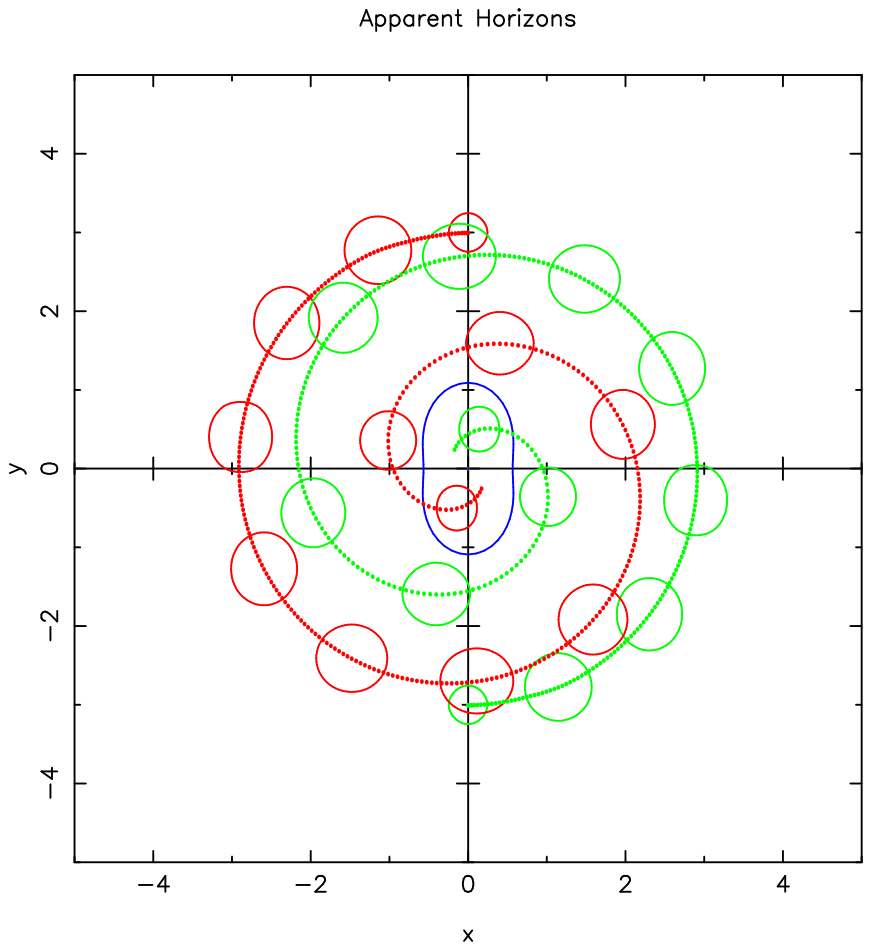


Merger time



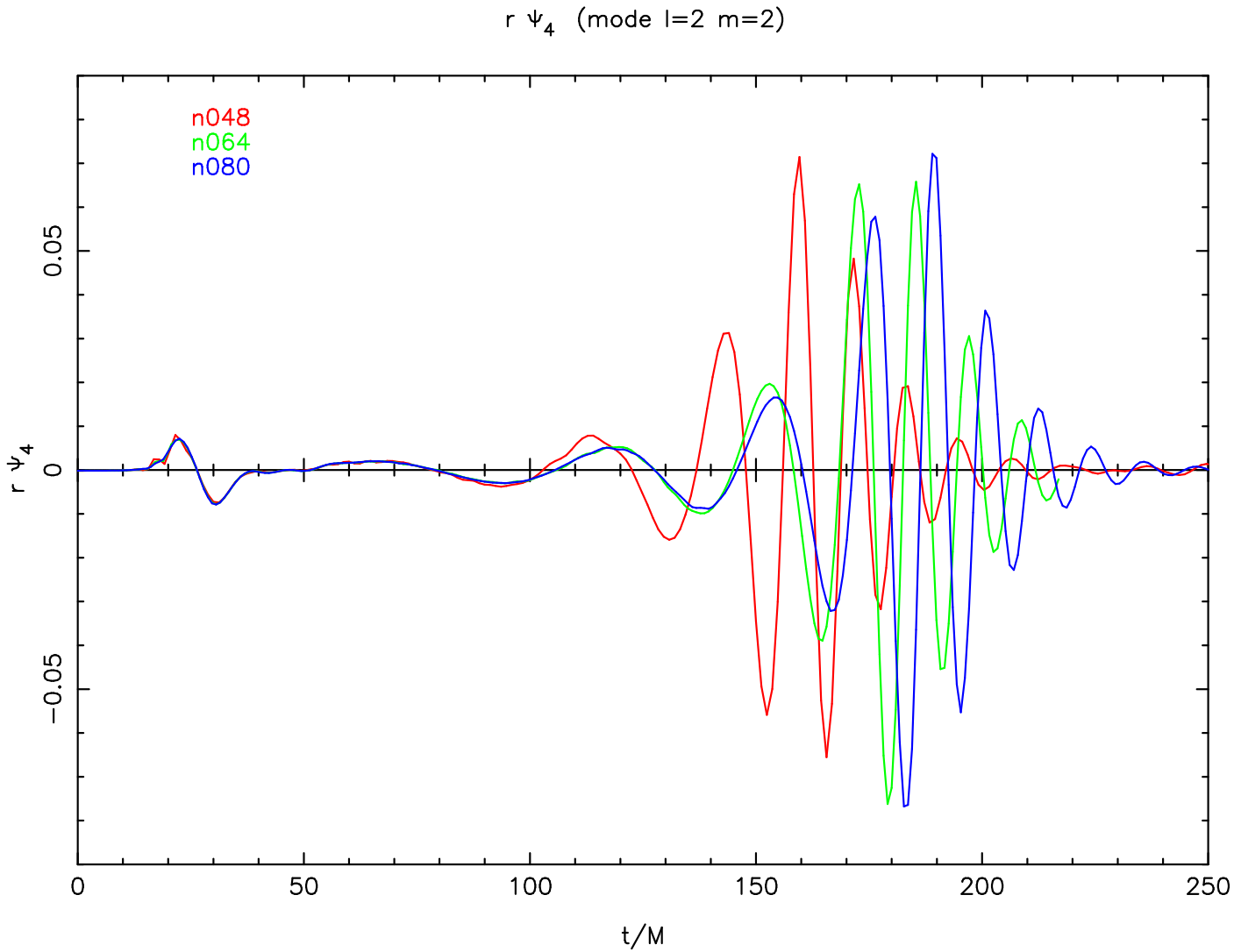
Apparent Horizon Finder. Fast Flow Method

Miguel Alcubierre's implementation ported to bam.

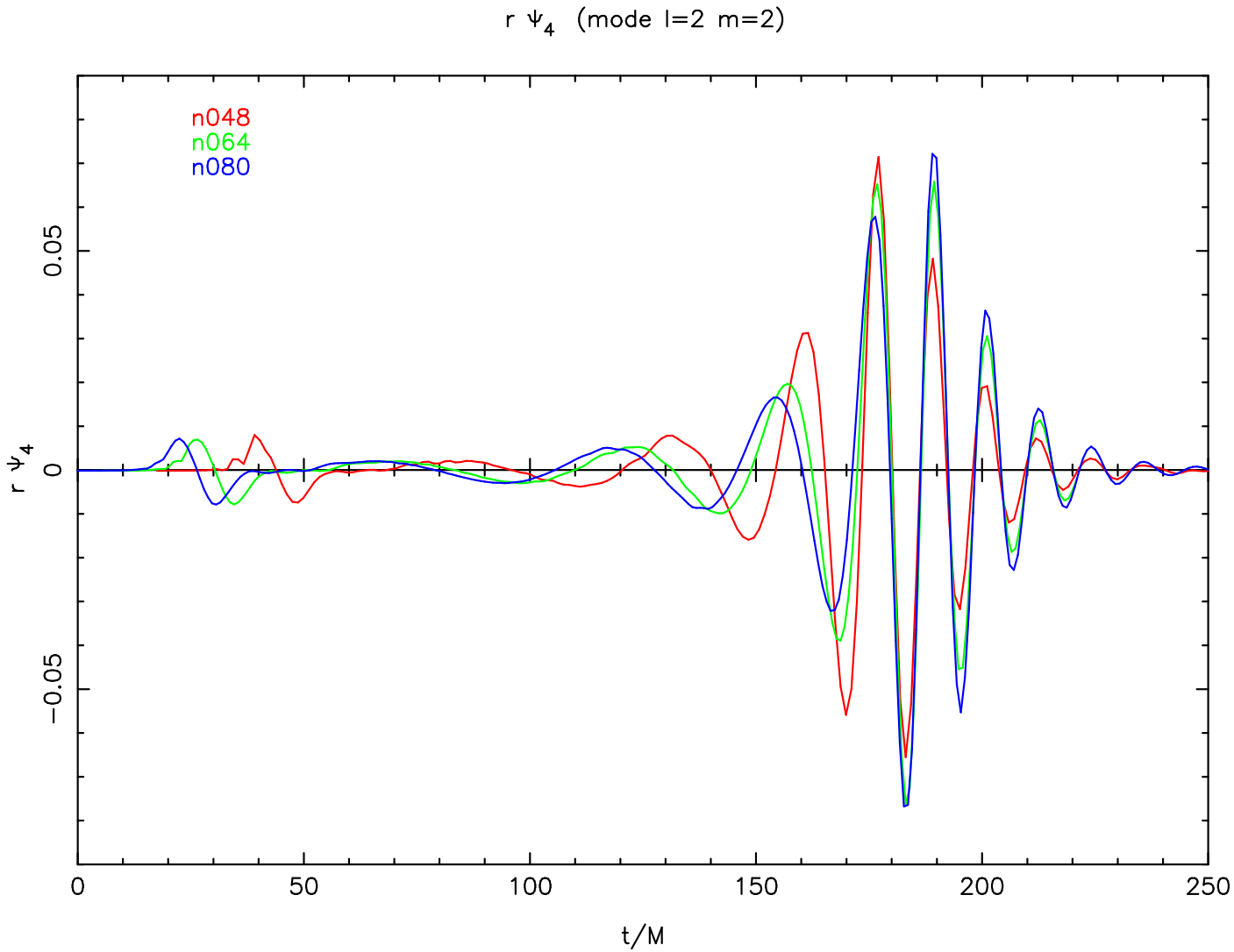


$R = 3$ lapse: $t_{0.3} = 111.82M$ AHF: $t_c = 117M$

Wave extraction. $r = 20M$

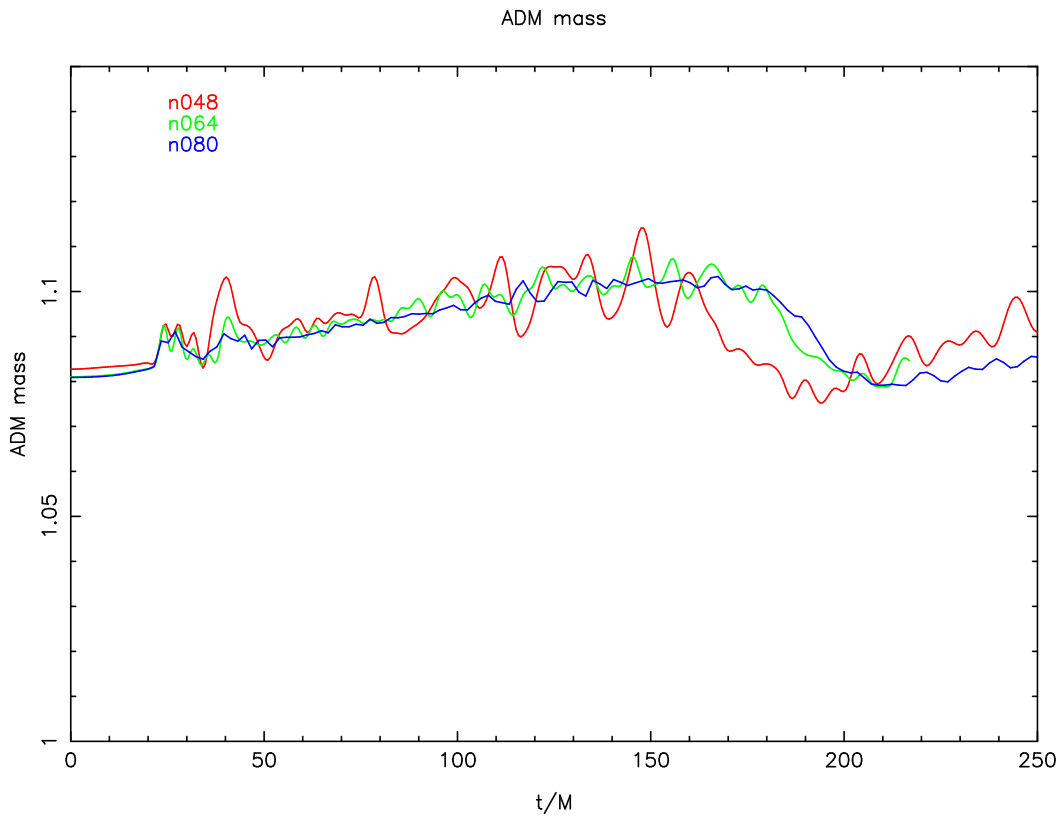


Wave extraction. $r = 20M$



ADM Mass at $r = 30$

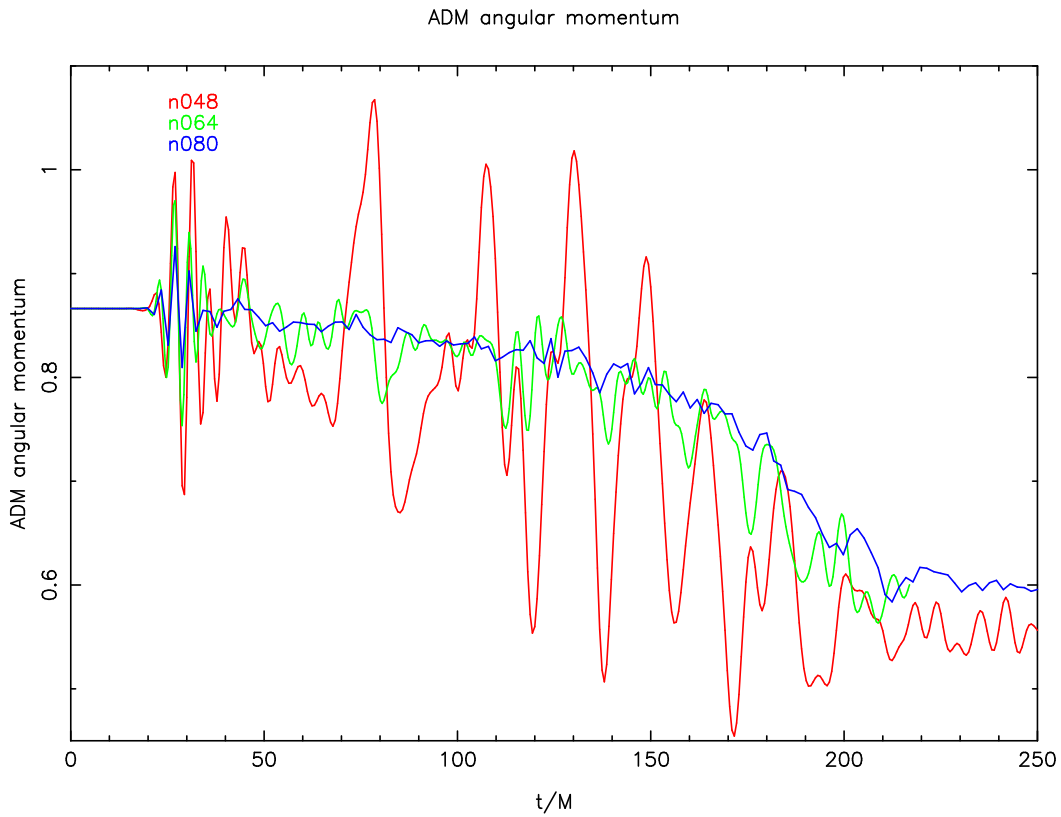
$$\text{“}E_{\rm ADM}(r=30)\text{”} \quad = \quad \frac{1}{16\pi}\int (\partial_m \gamma_{mn} - \partial_n \gamma_{mm})\, N^n dA$$



Initial Error $\sim$ 8% Radiated energy $\sim$ 3%.

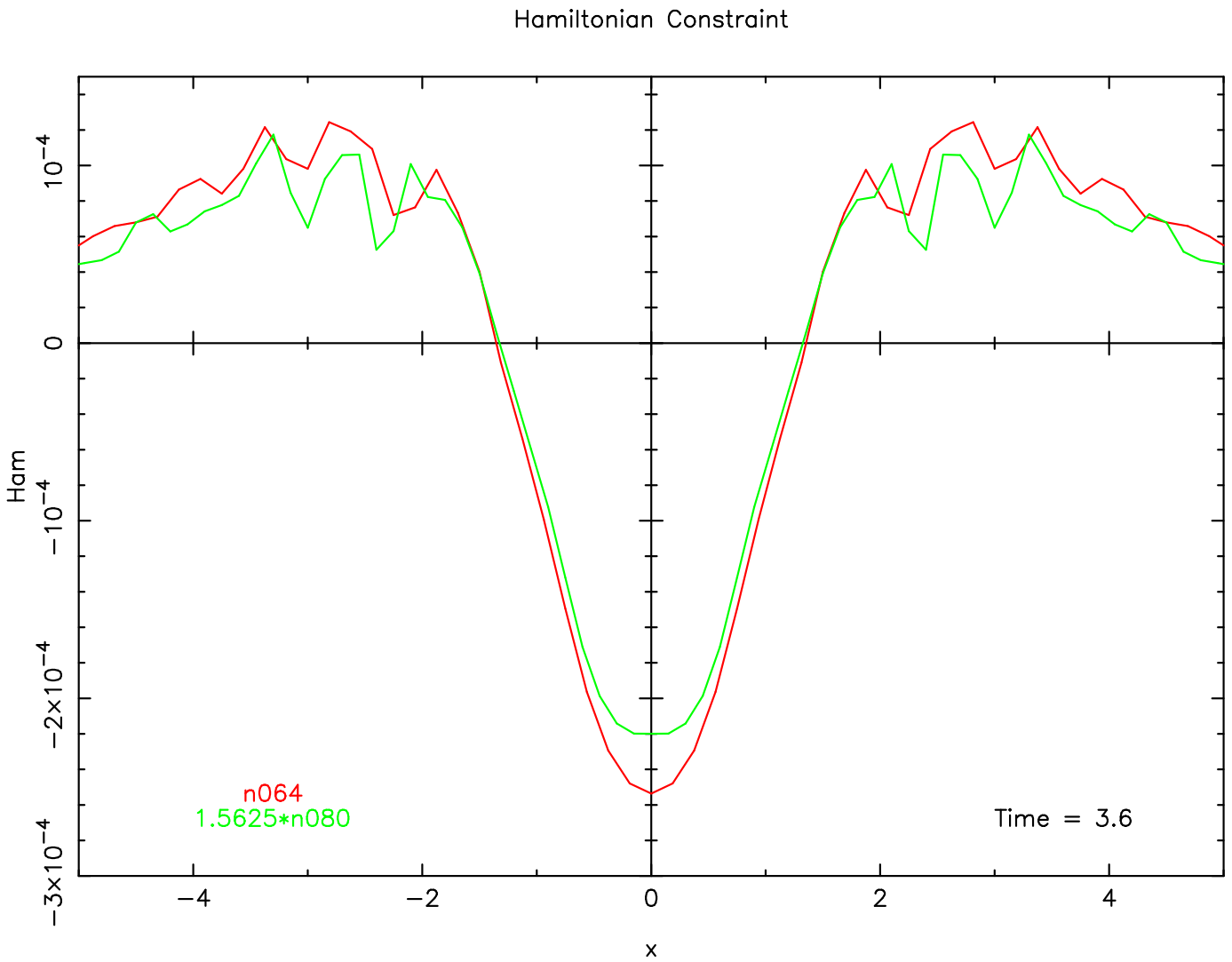
ADM Angular Momentum at $r = 30$

$$J_a = \frac{1}{8\pi}\epsilon_{am}{}^{n}\int x^m A^l{}_n N_l dA$$

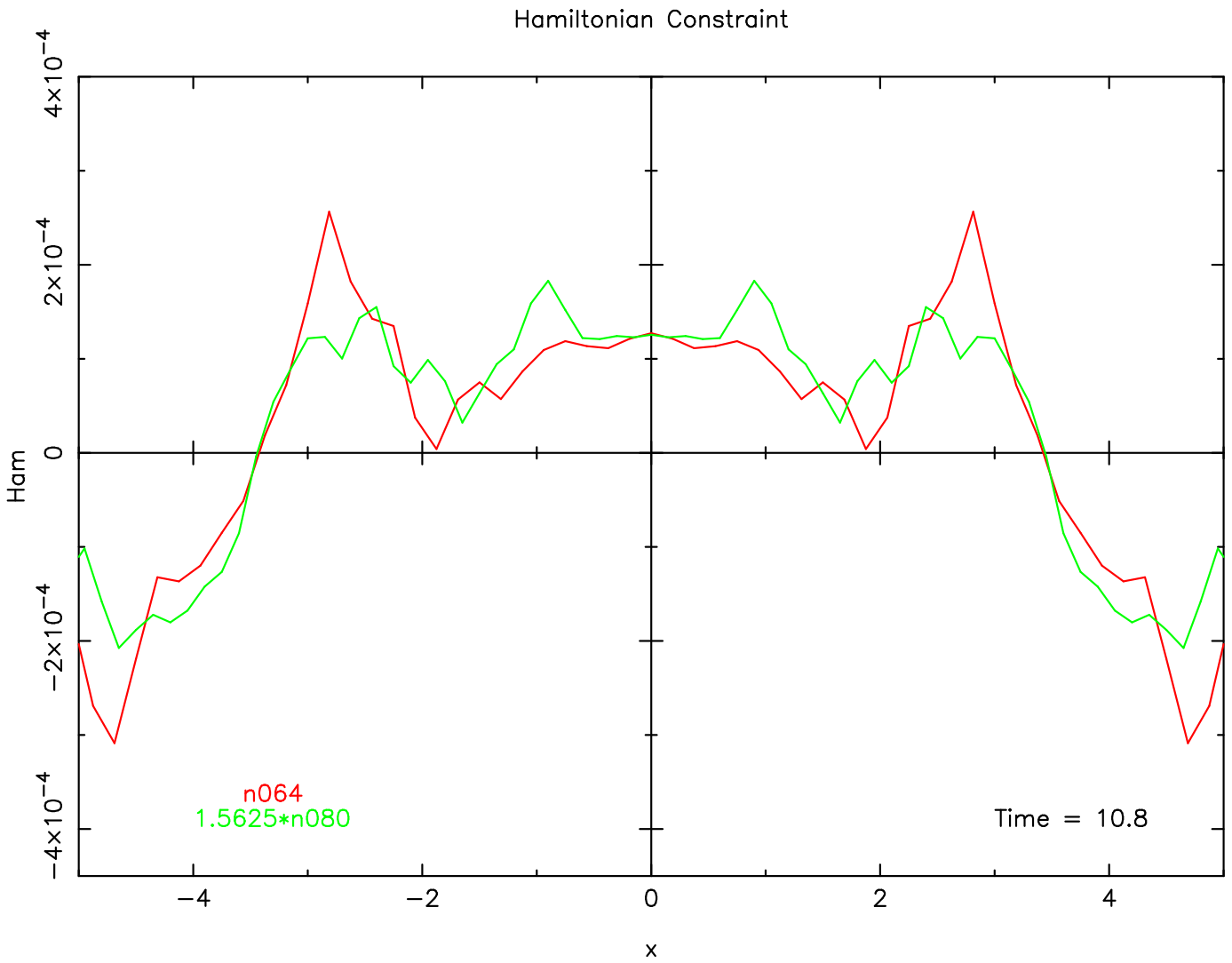


Radiated angular momentum $\sim$ 30%

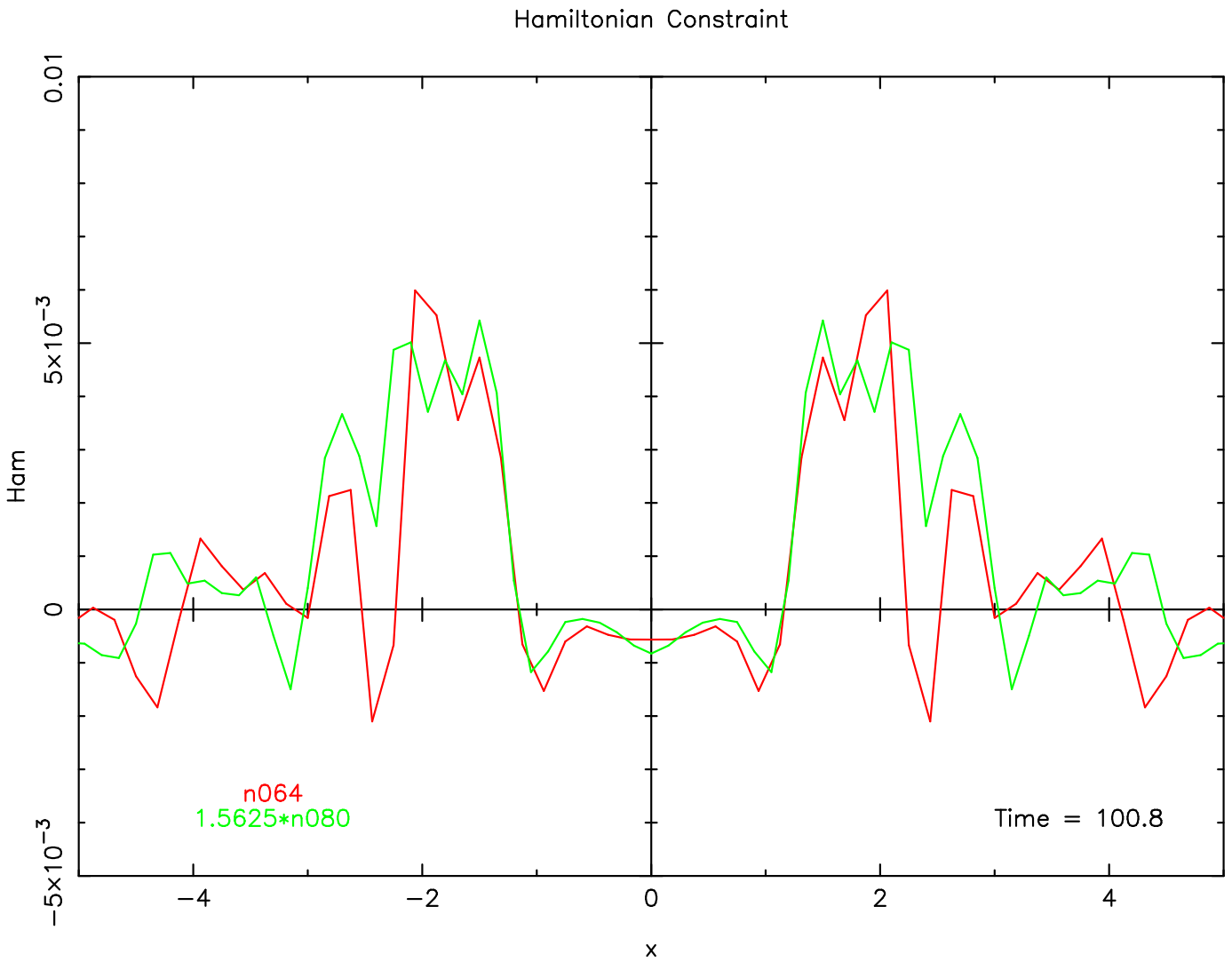
Hamiltonian Constraint at $t/M = 3.6$



Hamiltonian Constraint at $t/M = 10.8$



Hamiltonian Constraint at $t/M = 100.8$



Summary

- Moving boxes and mixed $2nd - 4th$ order in space and time.
- Convergence to $2nd$ order in:
  - constraints
  - merger time $\longrightarrow$ Early.
  - waves ?