The Quasi-Stationary Transition of Strange Matter Rings to a Black Hole

Hendrick Labranche, David Petroff and Marcus Ansorg
Friedrich-Schiller-Universität Jena

Part of project B1 of the SFB "Gravitationswellenastronomie" project leaders: Reinhard Meinel, Gernot Neugebauer

Introduction

- The extreme Kerr black hole is the only candidate for a parametric transition of stationary and axisymmetric (S&A) perfect fluid bodies to a black hole (Meinel).
- The transition to Kerr black holes in S&A spacetime has never been seen for spheroidal bodies but exists for **disk of dusts** (analytic solution, Neugebauer & Meinel) and **rings** (numerical solutions).
- For the class of ring fluids, numerical work (Ansorg et al.) has already been done with different equations of states (eos):
  - homogeneous density
  - polytropic eos
  - Fermi gas (Chandrasekhar eos)
- For strange matter, numerical work has already been done for the class of spheroidal bodies (Gourgoulhon et al.).

Strange Matter Rings: The Equation of State

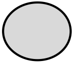
Strange matter is a fluid made of up (u), down (d) and strange (s) quarks. The equation of state (eos) is from the (very) idealized MIT bag model:

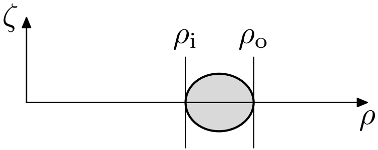
$$\epsilon = 3p + 4B$$

- ϵ is the energy density of the fluid
- p is the pressure
- B is a constant characterizing the quark confinement

In the Newtonian limit, $\epsilon \to$ constant.

Example of ring:





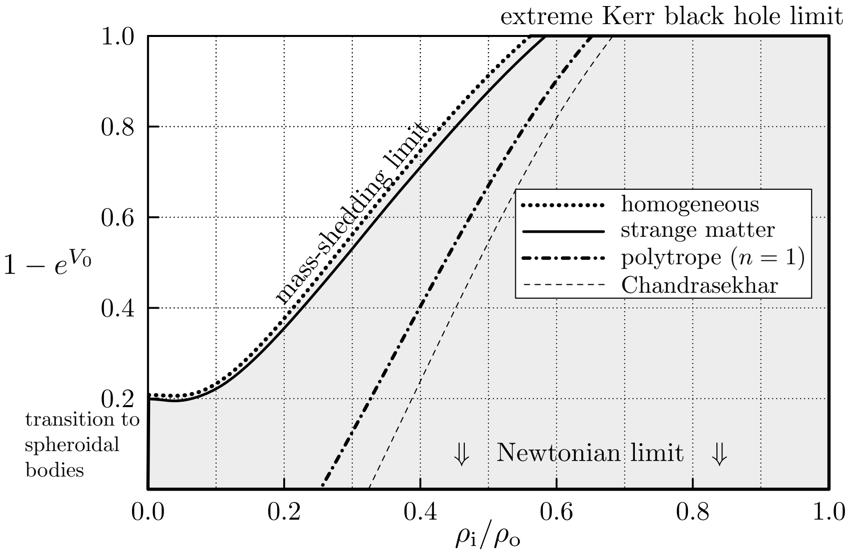


Figure: The parameter space for rings with a variety of eos in the (ρ_i/ρ_o)–$(1 - e^{V_0})$ plane.

From Metric to Multipole Moments

Metric for a stationary and axisymmetric spacetime:

$$ds^2 = e^{-2U}[e^{2k}(d\rho^2 + d\zeta^2) + W^2 d\varphi^2] - e^{2U}(ad\varphi + dt)^2$$

In vacuum, a conformal coordinate transformation gives

$$ds^2 = e^{-2U}[e^{2k'}(d\rho'^2 + d\zeta'^2) + \rho'^2 d\varphi^2] - e^{2U}(ad\varphi + dt)^2$$

with which we can define the Ernst potential $f = e^{2U} + ib$, $b = b(a, e^{2U})$.

$$\xi = \frac{1-f}{1+f}$$

We expand ξ at infinity on the positive ζ' axis

$$\xi(\rho' = 0, \zeta') = \sum_{n=0}^{\infty} \frac{m_n}{\zeta'^{n+1}}$$

The multipole moments defined by Geroch and Hansen are algebraic combinations of m_n.

The multipole moments P_n defined by Geroch and Hansen are algebraic combinations of m_n:

$$P_j = m_j \quad \text{for } j = 0, 1, 2, 3$$
$$P_4 = m_4 - \frac{1}{7}(m_2 m_0 - {m_1}^2) m_0$$
$$\vdots$$

For uniformly rotating bodies with angular velocity Ω, the multipoles P_n can be normalized as follows:

$$y_n = i(-2i\Omega)^{n+1} P_n.$$

such that an extreme Kerr black hole gives $y_n = 1$.

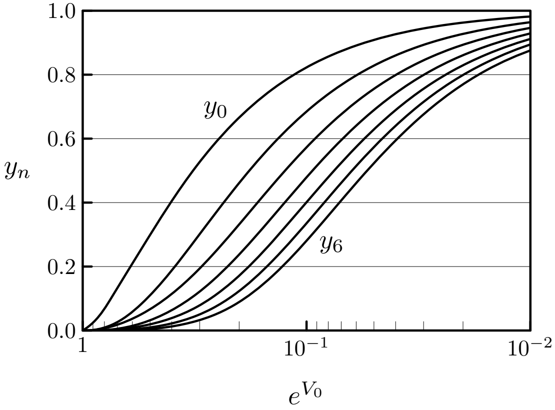


Figure: The multipoles y_n versus e^{V_0} for strange matter rings with $\rho_i/\rho_o = 0.7$

Reminder:
$e^{V_0} \to 1$ in the Newtonian limit, $e^{V_0} \to 0$ in the black hole limit.

We found that for the transition to an extreme Kerr black hole of a wide variety of rotating bodies, the multipole moments are quite similar.

Table: The multipole moments y_n for various configurations, all with $e^{V_0} = 10^{-2}$.

	y_0	y_1	y_2	y_3	y_4
s.m. $(r_i/r_o = 0.6)$	0.982	0.964	0.947	0.930	0.913
s.m. $(r_i/r_o = 0.7)$	0.981	0.963	0.945	0.928	0.910
s.m. $(r_i/r_o = 0.8)$	0.981	0.962	0.943	0.925	0.907
hom. $(r_i/r_o = 0.7)$	0.981	0.963	0.945	0.927	0.910
pol. $(r_i/r_o = 0.7)$	0.982	0.965	0.948	0.931	0.914
rel. disc of dust	0.984	0.969	0.953	0.938	0.924

Multipoles: Ring vs Black Hole

The parameter e^{V_0} cannot be defined for a Kerr Black Hole. But the multipoles can be represented as follows:

$$J = \frac{4\Omega_H M^3}{1 + 4\Omega_H^2 M^2},$$

$$P_n^{(\mathrm{Kerr})} = M(iJ/M)^n,$$

$$y_n = i(-2i\Omega)^{n+1} P_n,$$

$$y_n^{(\mathrm{Kerr})}(y_0) = y_0 \left(\frac{2y_0^2}{1 + y_0^2} \right)^n .$$

Near the extreme Kerr limit, an interesting characteristic is the derivative

$$\frac{dy_n}{dy_0}(y_0 = 1) = n + 1.$$

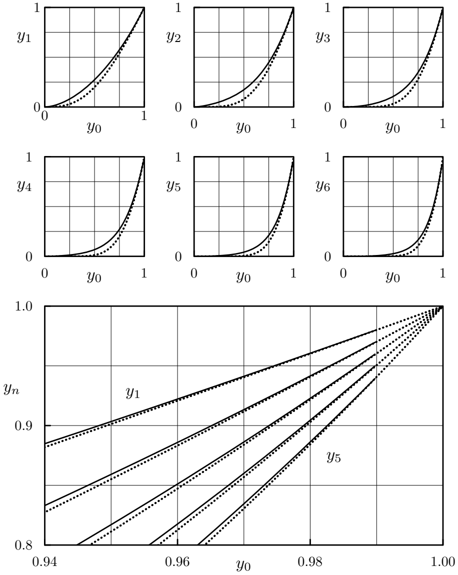


Figure: y_n versus y_0 for strange matter rings with $\rho_i/\rho_o = 0.7$ (solid lines) and the sequence of Kerr solutions (dotted lines).

Can we find an analytic explanation for this behaviour of the multipoles?

$$\frac{dy_n}{dy_0}(y_0 = 1) = n + 1.$$

From the analytic potentials of the disk of dust, we confirm that:

$$\frac{dy_1}{dy_0}(y_0 = 1) = 2.$$

We are working now on the other multipoles.
In general, can we find this behaviour directly from the Einstein Equations?

Throat Geometry

A throat geometry appears for bodies near the extreme Kerr black hole limit. In the limit:

- An infinitely long throat separates an ‘inner world’ from an ‘outer world’.
- The inner world contains the ring and is not asymptotically flat.
- The outer world is identical to the asymptotically flat extreme Kerr spacetime.

In the equatorial plane, the proper distance δ from the point $\rho = 0$ to the point $\rho = \tilde{\rho}$ is

$$\delta = \int_0^{\tilde{\rho}} \sqrt{g_{\rho\rho}}\Big|_{\zeta=0} \, d\rho.$$

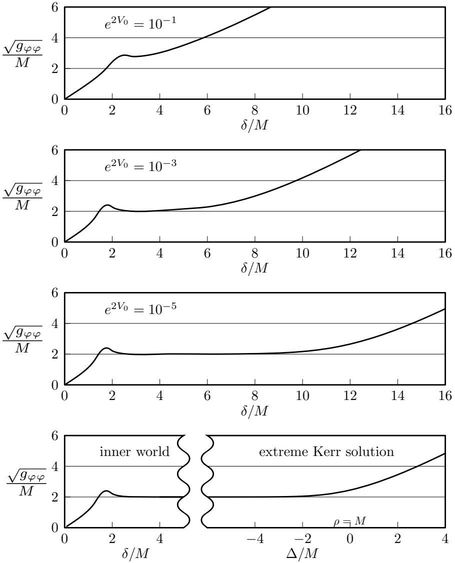


Figure: $\sqrt{g_{\varphi\varphi}}$ in the equatorial plane vs proper distance δ, both normalized with respect to the mass M, for rings with different e^{2V_0}.

Summary

- The transition of strange matter rings to a black hole exists. The extreme Kerr black hole results.
- Conjecture: The multipole moments of every rotating body approach the black hole limit with a unique behaviour:

$$\frac{dy_n}{dy_0}(y_0 = 1) = n + 1.$$

- Near the extreme black hole limit, an infinitely long throat appears, isolating the body from the "outer world".