How far can we trust the PN spin?

Long numerical waveforms from spinning binaries

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Results from
Achamveedu Gopakumar, MH, Sascha Husa, Bernd Brügmann, arXiv:0712.3737
and
MH, Sascha Husa, Bernd Brügmann, Achamveedu Gopakumar, arXiv:0712.3787

The big picture: detecting gravitational waves

General motivation:

- Gravitational-wave detection requires accurate theoretical templates
- For black-hole binary inspiral, most current templates are based on PN theory
- Are these good enough?
  - For most of the inspiral, hopefully *Yes* — but for how much?
  - For merger, *No* — PN expansion breaks down and waveforms are "cut off"
- One solution:
  - Numerically produce long waveforms in full GR
  - Determine for how long the PN inspiral waveforms are accurate enough
  - Stitch on the NR waveforms after that.

Technical caveats:

- "Accurate enough" depends on the source, the detector, and what you want to measure!
- In practice, comparisons really only make sense in relation to a given detector's noise curve and the total mass of the source (i.e., the frequency range of the waveform).

But first we need to make a direct comparison anyway!

The desired final product: a hybrid waveform

Equal-mass non-spinning example:

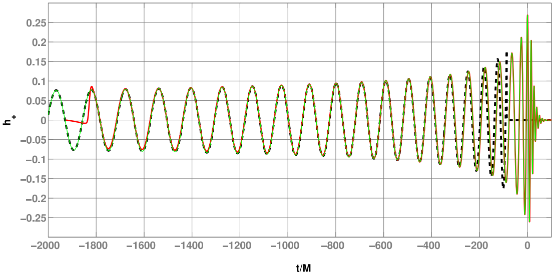


Figure: P. Ajith, *et. al.*, arXiv:0710.2335

To do this, you want to know in what region PN and NR agree, and by how much

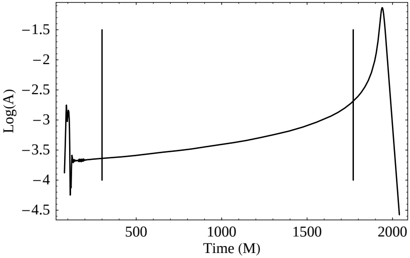
What we compare, and how

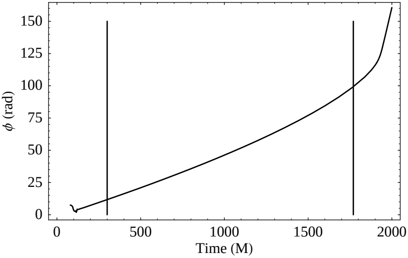
Wave extraction from numerical results gives Ψ_4 ($= \ddot{h}$).

We focus on the ($l = 2, m = 2$) mode.

We can decompose this into amplitude and phase,

$$r\Psi_{4,22} = A(t)e^{-i\phi(t)}.$$





Amplitude and phase from PN approximants

Conveniently, the PN approximants also consider phase and amplitude separately!

Phase can be given up to 3.5PN order.

We compare with three PN approximants for the phase:

- TaylorT1 and TaylorT4 are expansions in terms of $x = (M\omega)^{2/3}$.
- TaylorEt is an expansion in terms of the binding energy of the binary.
- All three give ODEs for $\phi(t)$, which are solved numerically.

Amplitude is often given at leading order (quadrupole, "restricted" PN)

$$A_{\rm restricted} = \frac{(M\omega/2)^{2/3}}{r}$$

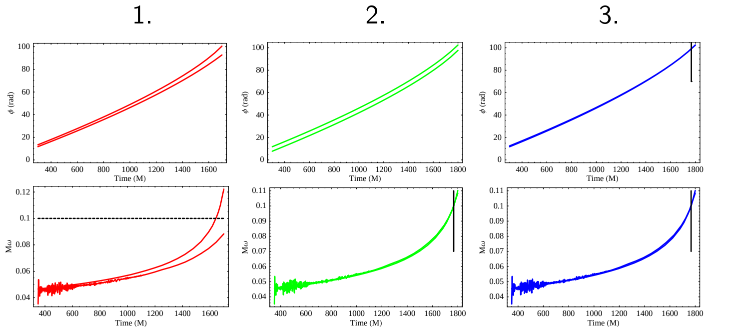
Higher-order amplitude corrections are known for some cases
(e.g., up to 3PN for equal-mass nonspinning binaries)

Lining up the phase

When comparing $\phi(t)$, we need to line up the PN and NR curves.

We can apply an arbitrary constant time and phase shift.

1. Choose a frequency ω_0 at which we want PN and NR to agree
2. Time-shift so that the frequencies agree at the same time, $\omega_{PN}(t_0 + \Delta t) = \omega_{NR}(t_0) = \omega_0$.
3. Phase-shift so that phases agree at that time: $\phi_{PN}(t_0 + \Delta t) + \Delta\phi = \phi_{NR}(t_0)$.



Comparison choices

How do we choose ω_0?

- We expect PN to be more accurate at low frequencies
- But NR results are less noisy and more accurate at high frequencies
- We generally choose $M\omega_0 = 0.1$. (About half the PN ISCO frequency; ~ 1.5 orbits before merger.)

Now measure the accumulated phase error as we go back in time.

There are many alignment and comparison options — beware of rash conclusions!

Amplitude

The PN amplitude as a function of *frequency* is independent of the approximant used for the phase.

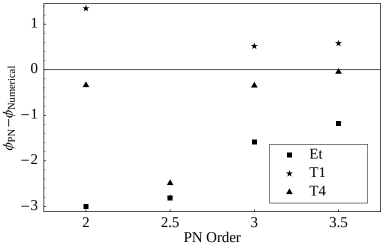
We therefore compare $A_{PN}(\omega)$ with $A_{NR}(\omega)$.

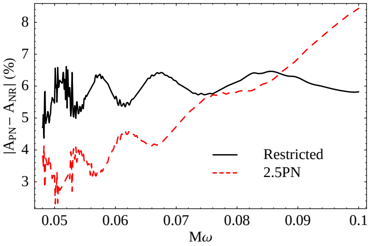
PN-NR comparison: equal-mass nonspinning binaries

Over $\approx$ 14 cylces up to $M\omega = 0.1$, phase agreement is within 1.5 radians.

Amplitude disagreement with restricted PN is about 6%.

With 2.5PN amplitude, the disagreement is about 3% at low frequencies.





See arXiv:0706.1305 (June 2007),
and arXiv:0712.3737 (December 2007, comaprison with TaylorEt approximant).

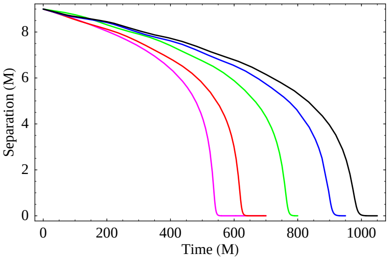
Onwards: spinning binaries

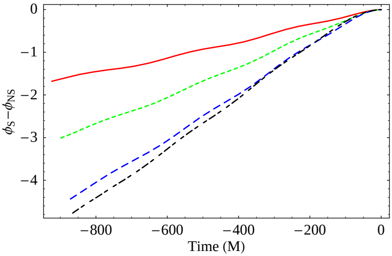
Equal mass, equal spins, *parallel* to orbital angular momentum.

Consider $S_i/M_i^2 = 0.25, 0.5, 0.75, 0.85$.

This is the "orbital hang-up" case: the spin slows down the merger.

Put another way, the phase evolution is quite different
(compare with the nonspinning case)





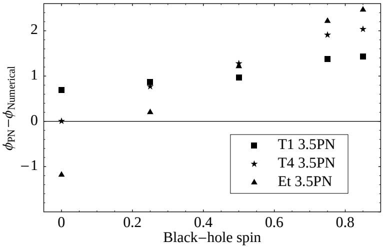
PN-NR comparison: spinning binaries

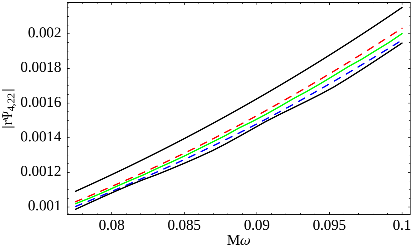
Measure the phase disagreement over the 10 cycles up to $M\omega = 0.1$.

Look at the 3.5PN case (even though not all spin terms are known beyond 2.5PN).

- For all approximants, phase disagreement is less than 2.5 radians.
- For TaylorT1, it is roughly constant at $\Delta\phi \approx 1$ radian.
- Recall: the phase difference between spinning and nonspinning is much larger $\Rightarrow$ PN is capturing the spin effects well!

Restricted amplitude disagreement grows to about 12% for $S_i/M_i^2 = 0.85$.



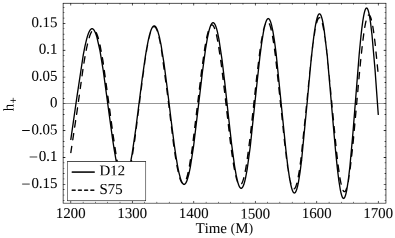


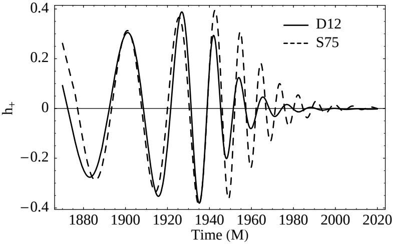
Detection of spinning binaries

During inspiral, it's difficult to distinguish spinning from nonspinning

(Spinning binary = nonspinning binary with a different mass.)

But at the merger, the two are clearly distinct.





Overlap integral over inspiral: 0.99

Overlap integral over merger: 0.90.

Summary, conclusions and future directions

Summary and conclusions

- Nonspinning, equal-mass: $\Delta\phi < 1.5$ rad for 14 cycles before $M\omega = 0.1$. Restricted amplitude disagreement is about 6%.
- For the orbital hang-up case, $\Delta\phi$ is a little worse, but not much. Restricted amplitude disagreement rises to up to 12%.
- For many cases, this should be good enough to produce hybrid waveforms.
- PN phase evolution captures spin effects well (for the orbital hangup case).
- From PN theory, we need higher-order amplitude corrections.
- Distinguishing between spinning and nonspinning binaries is *much* easier if the inspiral *and* merger are detected.

Future work

- Make similar comparisons with detector noise
- Test these waveforms in data analysis pipelines
- Extend these studies to the rest of parameter space!