Inspiral gravitational wave templates: role of the PN-expansion parameter

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Outline

- Post-Newtonian accurate GW phasing for inspiralling compact binaries

  Why widely employed $x = (G\, m\, \omega / c^3)^{2/3}$*-based* templates require a closer inspection ?

- Adiabatic & non-adiabatic PN-accurate inpiral templates, $h_{\times,+}(t)$, for comparable mass compact binaries

- Comparing the performances of adiabatic & non-adiabatic $h_{\times,+}(t)$ & probing its implications for LIGO/VIRGO

References

1. M. Tessmer & AG, arXiv:0712.3199 [gr-qc]; to appear in Phys. Rev. D
2. AG, arXiv:0712.3236 [gr-qc].
3. AG, M. Hannam, S. Husa and B. Brugmann, arXiv:0712.3737 [gr-qc]; to appear in Phys. Rev. D
4. M. Hannam, S. Husa, B. Brügmann and AG, **http://xxx.lanl.gov/abs/0712.3787**; to appear in Phys. Rev. D
5. S. Bose, AG & M. Tessmer, arXiv:0807.2400 [gr-qc]
6. M. Tessmer & AG (2008); MS in preparation

PN approximation

- For LIGO/VIRGO applications, one needs to tackle two problems
- i) Problem of finding PN-accurate equations of motion $\ddot{X}$
  & ii) Problem of computing PN-accurate gravitational-wave luminosity $\mathcal{L}$
  & polarizations $h_{\times,+}$
  *n*PN order: corrections of order
  $$\left(\frac{v}{c}\right)^{2n} \sim \left(\frac{Gm}{rc^2}\right)^n$$
  to the Newtonian gravity

$\ddot{X}$	N	1PN	2PN	2.5PN	3PN	3.5PN	4PN	4.5PN	5PN	5.5PN	6PN
$\mathcal{L}$	—	—	—	N	—	1PN	1.5PN	2PN	2.5PN	3PN	3.5PN
$h_{\times,+}$	—	—	N	0.5N	1PN	1.5PN	2PN	2.5PN	3PN		

PN quantities

Blanchet, Damour, Schäfer & their collaborators, after many years of computations, provided FOUR valuable expressions for compact binaries in PN accurate circular orbits

- 3PN accurate dynamical (orbital) energy $\mathcal{E}(x)$ as a PN series in $x = (G\, m\, \omega_{3\mathrm{PN}}/c^3)^{2/3}$
  $\omega_{3\mathrm{PN}}(t)$ the 3PN accurate orbital angular frequency
  Damour, Jaranowski & Schäfer (2001)

- 3.5PN accurate expression for GW energy luminosity $\mathcal{L}(x)$
  Blanchet et.al (2002) & (2005)

- 3PN amplitude corrected expressions for $h_+(t)$ & $h_\times(t)$ in terms of the orbital phase $\phi(t)$ and $x(t)$
  Blanchet et.al (2008)

LAL Routines

The LSC Algorithms Library (LAL) employs these inputs to construct various types PN-accurate inspiral search templates

TaylorT1 Damour, Iyer & Sathyaprakash (2001)

$$h(t) \propto \left(\frac{G\, m\, \omega(t)}{c^3} \right)^{2/3} \cos 2\, \phi(t)\,, \tag{1}$$

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TaylorT1 $h(t)$

$$\frac{d\phi(t)}{dt} = \omega(t)\,; \quad \frac{d\,\omega(t)}{dt} = -\mathcal{L}(\omega) \Big/ \frac{d\mathcal{E}}{d\omega}\,, \tag{2}$$

To compute TaylorT1 3.5PN $h(t)$, one needs 3.5PN accurate GW luminosity $\mathcal{L}(\omega)$ & 3PN orbital energy $\mathcal{E}$

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TaylorT4 3.5PN $h(t)$: Very close to NR inspiral $h(t)$, but not in LAL

$$\frac{d\phi(t)}{dt} \equiv \omega(t)\,;\; \frac{d\,\omega(t)}{dt} = \frac{96}{5} \left(\frac{G\,\mathcal{M}\,\omega}{c^3} \right)^{5/3} \omega^2 \Big\{ 1 + \mathcal{O}(\nu) + \mathcal{O}(\nu^{3/2}) + \mathcal{O}(\nu^2) + \mathcal{O}(\nu^{5/2}) + \mathcal{O}(\nu^3) + \mathcal{O}(\nu^{7/2}) \Big\}\,, \tag{2}$$

$\mathcal{M}$ being the chirp mass & $\nu = 1/c^2$, a PN ordering parameter

Issues in GW phasing:Ia

- GW phase evolution begins at the absolute 2.5PN order for all sorts of PN-accurate LSC inspiral templates

- Such $h_{\times,+}(t)$ are usually refered to as 'adiabatic templates' !
Nomenclature due to Ajith *et.al.* [**http://arxiv.org/abs/gr-qc/0412033**]

- They model GWs from compact binaries inspiraling under PN-accurate reactive dynamics *along exact circular orbits*
This is why $\phi_{\rm GW}(t)$ *begins at 2.5PN order*

- According to **AIRS**, adiabatic templates do not explicitly reflect the fact that there are 1PN & 2PN contributions to the orbital dynamics of a compact binary

Issues in GW phasing:Ib

- Widely employed PN-accurate inspiral templates are essentially given by one diff-eqn: $\frac{d^2\phi}{dt^2}(\dot{\phi} = \frac{d\phi}{dt}, m, \eta)$

  This is due to the use of $\dot{\phi} \equiv \omega$ as a PN-expansion parameter

- Limits of integration are the same for Newtonian & any PN-accurate inspiral templates & these are independent of η

- The use of $d\phi/dt = \omega$ as the usual PN expansion parameter for the circular inspiral is justified by the argument that it defines the instantaneous orbital angular frequency & GW frequency

  The above argument is NOT extendable to the case of eccentric inspiral; however small may be the orbital e_t.

Non-adiabatic & gauge-invariant PN-accurate TaylorET $h(t)$

The restricted 3.5PN accurate TaylorET $h(t)$ is given by

$h(\hat{t}) \propto \tilde{\mathcal{E}}(\hat{t}) \cos 2\,\phi(\hat{t})$

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$$\frac{d\phi}{d\hat{t}} = \tilde{\mathcal{E}}^{3/2}\left\{1 + \tilde{\mathcal{E}}\left[..\right] + \tilde{\mathcal{E}}^{2}\left[..\right]\right\}, \qquad (3)$$

$$\frac{d\tilde{\mathcal{E}}}{d\hat{t}} = \frac{64}{5}\,\eta\,\tilde{\mathcal{E}}^{5}\left\{1 + \tilde{\mathcal{E}}\left[..\right] + ...\tilde{\mathcal{E}}^{7/2}\left[..\right]\right\},$$

$\tilde{\mathcal{E}}/2$ is the dimensionless non-relativistic energy per unit reduced mass

A.Gopakumar, arXiv:0712.3236 [gr-qc]

Issues in GW phasing: II

- PN-accurate TaylorEt approximant models GWs from compact binaries inspiraling under PN-accurate reactive dynamics along *PN-accurate circular orbits*

- Therefore, the new approximant *explicitly* incorporates the secular contributions to GW phase evolution appearing at the 1PN, 2PN orders before radiation reaction kicks in at 2.5PN order

- The construction of TaylorEt $h(t)$ is influenced by a prescription that provided accurate & efficient $h_{\times,+}(t)$ for comparable mass compact binaries inspiralling along PN-accurate eccentric orbits (DGI-04,KG-06)

Issues in GW phasing: III

- The use of the standard energy balance argument implies that various inspiral templates model GWs from compact binaries *evolving along a sequence of circular orbits* under the action of gravitational radiation reaction.

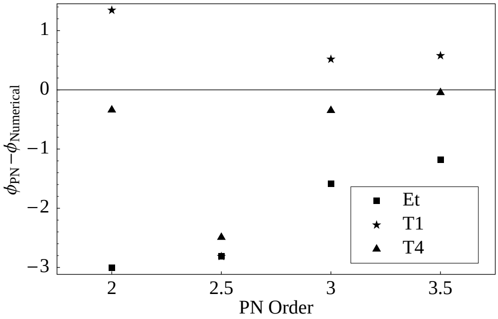
- For the various adiabatic templates, the above *circular orbits are exact*

- For the Taylor Et approximant, these *circular orbits are 3PN-accurate*

- In other words, there are two PN-accurate diff-eqns & the limits of integration require 3PN accurate conservative dynamics

Late time GW phase evolution under TaylorEt approximant is also fairly accurate w.r.t Numerical-Relativity based $\phi_{\rm GW}(t)$

arXiv:0712.3737



Issues in GW phasing: IV

- Via fitting factor studies, we probed if the TaylorEt (3.5PN) signals for non-spinning comparable mass compact binaries can be *effectually & faithfully* searched with TaylorT1, TaylorT4, and TaylorF2 (3.5PN) templates in LIGO/Virgo interferometers.
  S. Bose, A.Gopakumar & MT(http://arxiv.org/abs/0807.2400)

- For equal-mass binaries, adiabatic templates are neither effectual nor faithful w.r.t TaylorEt h(t)

- For unequal-mass binaries, adiabatic templates are rather effectual, but highly unfaithful w.r.t TaylorEt h(t)

- TaylorEt arises from the circular limit of GW phasing for eccentric binaries, detailed in Damour-Gopakumar-Iyer (2004).
  The associated $h_{\times,+}(t)$ are non-adiabatic in nature

Eccentric GW phasing:I

- **Restricted $h_\times$ for eccentric binaries**
  $h^0_\times(r,\phi,\dot r,\dot\phi) = -2\frac{G\,m\,\eta\,C}{c^4\,R'}\left\{\left(\frac{G\,m}{r} + r^2\,\dot\phi^2 - \dot r^2\right)\sin 2\phi - 2\dot r\,r\,\dot\phi\,\cos 2\phi\right\}$
  where $C = \cos i$.
- To construct 'search templates', we require PN accurate $h_{+,\times}$ supplemented by explicit expressions describing the temporal evolution of the PN accurate relative motion, *i.e.* $r(t)$, $\phi(t)$, $\dot r(t)$, & $\dot\phi(t)$
- coordinate dependent for eccentric inspiral & we want to be as 'analytical' as possible.
- We employ an improved method of variation of constants to incorporate the radiation reaction to obtain *phasing relations'* $r(t)$, $\phi(t)$, $\dot r(t)$, & $\dot\phi(t)$

Eccentric GW phasing:II

A summary of DGI-04 approach:

- Using 3PN-accurate Keplerian-type parametric solution to 3PN-accurate conservative EOM, we obtain

$$\frac{d\phi}{dt} = \dot{\phi}_{\rm 3PN}(\mathcal{E}, e_t, t) \quad (4)$$

- Using 2PN-accurate far-zone energy and angular momentum fluxes for eccentric binaries, we compute

$$\frac{d\mathcal{E}}{dt} = \dot{\mathcal{E}}_{\rm 2PN}(\mathcal{E}, e_t) \quad (5)$$
$$\frac{de_t}{dt} = \dot{e}_{t\,\rm 2PN}(\mathcal{E}, e_t)$$

- Solving above Eqs. numerically, we obtain GW phase evolution for eccentric binaries

- Circular limit of above leads to non-adiabatic TaylorEt approximant

Eccentric GW phasing:III

Eccentric phasing: Blanchet's way; **arxiv:0806.1037**

- Compute $\frac{d\phi}{dt}$ as in our way

$$\frac{d\phi}{dt} = \dot{\phi}_{\rm 3PN}(\mathcal{E}, e_t, t) \quad (6)$$

- Define via orbital-averaging $< \frac{d\phi}{dt} > \equiv \omega(\mathcal{E}, e_t)$ & re-express Eq. (6) as

$$\frac{d\phi}{dt} = \dot{\Phi}_{\rm 3PN}(\omega, e_t, t) \quad (7)$$

- Using 2PN-accurate far-zone energy and angular momentum fluxes for eccentric binaries, compute

$$\frac{d\omega}{dt} = \dot{\omega}_{\rm 2PN}(\omega, e_t) \quad (8)$$

$$\frac{de_t}{dt} = \dot{e}_{t\,\rm 2PN}(\omega, e_t)$$

- Circular limit of above leads to adiabatic TaylorT4 approximant

Eccentric GW phasing:IV

- The LSC regularly employs 2PN-accurate ‘adiabatic circular templates’, TaylorF2 ($\equiv$ TaylorT4) to search for inspiral GWs from comparable mass compact binaries

- We observe that such templates can not capture GWs from compact binaries inspiralling along PN-accurate eccentric orbits, constructed, following DGI-approach, in an effectual’ & faithful manner, even when $e_t^i \sim \leq 10^{-4}$

  M. Tessmer & A.G, (MS in preparartion)

Issues in GW phasing: V

- The LSC employed Martel & Poisson (2000) paper to ignore GWs from ICBs having tiny orbital e_t.

- We demonstrated why the LSC should NOT trust the above paper too much!
  M. Tessmer & A.G, (PRD, 2008)

- Can one really trust the astrophysical conclusions of the following LSC paper ?
  Implications for the Origin of GRB 070201 from LIGO Observations ?
  The LSC employed 2PN-accurate x-based templates in the above MS.

- More investigations are required