Black-hole-binary simulations: what are they good for?

Verifying the consistency of black-hole-binary waveforms for gravitational-wave detection

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Results from Phys. Rev. D, 79, 084025 (2009), arXiv:0901.2437,
involving researchers at
AEI Potsdam, Caltech, Cornell, FSU Jena, Georgia Tech, Maryland, NASA-Goddard, Penn State, Cork, UIB Palma

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Using numerical BBH waveforms for GW astronomy

GW detection has been the number one motivation for BBH simulations

If we are to really use NR results for GW detection, we need to know

- How accurate the waveforms need to be
- How "long" they need to be
- How to connect the late inspiral/merger/ringdown to PN inspiral waveforms
- How much of the BBH parameter space needs to be sampled
- How to "interpolate" the NR results to produce template banks across the entire parameter space

In this talk I summarize work that addresses the first question, namely:

Are current simulations accurate and reliable enough for GW astronomy applications

How accurate we *think* our waveforms are

All good NR waveforms come with uncertainty estimates

For example, `BAM` simulations of the equal-mass nonspinning case:

- Cover $\approx$18 GW cycles before merger
- Phase uncertainty: 0.1 rad during inspiral, 1.0 rad during merger/ringdown
- Amplitude uncertainty 4% during inspiral, 6% during merger/ringdown.

What do these numbers mean for GW detection?

What about parameter estimation?

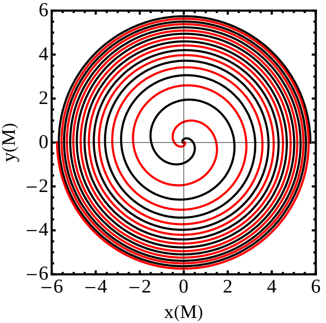
Are they consistent with other codes,
that use different gauges, wave extraction techniques, initial data,
formulations of Einstein's equations, numerical methods... ?

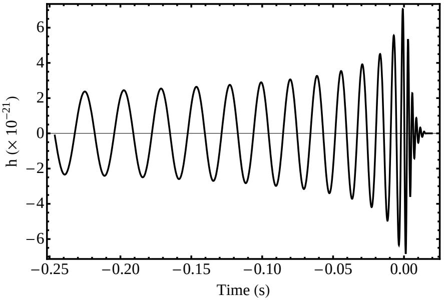
These numbers alone tell us nothing about how useful this waveform is for GW applications

The Samurai project

Using results from five codes, compare

- the last $1000M$ of inspiral, plus merger and ringdown
- for an equal-mass nonspinning binary following quasi-circular inspiral
- using measures relevant to both numerical relativity and GW data analysis





Right: the gravitational-wave strain from an optimally-oriented 60 $M_\odot$ equal-mass nonspinning black-hole binary located 100 Mpc away from the detector. The waveform covers about six orbits, or twelve GW cycles, before merger.

The Samurai codes and waveforms

Four moving-puncture codes: `BAM`, `CCATIE`, `Hahndol`, `MayaKranc`

- Moving-puncture method: modified BSSN/1+log slicing/$\tilde{\Gamma}$-driver shift
- Bowen-York-puncture initial data
- Finite-differences with mesh-refinement grids
- Ψ_4 or Zerilli wave extraction
- Simulations differ in details of grid structure, resolutions, finite-difference orders.

One generalized-harmonic code: `SpEC`

- Variant of generalized-harmonic formulation
- Conformal-thin-sandwich excision initial data
- Multi-domain pseudo-spectral method in space.
- Ψ_4 wave extraction

Two waveforms of note

SpEC

- Involves the largest number of differences from the other four codes
- Claims the lowest uncertainty estimate in the numerical waveform
- All comparisons are performed between `SpEC` and the others.

Hahndol

- The oldest waveform included in the analysis
- Claims the *highest* uncertainties...
- Phase uncertainty 2.4 rad during inspiral, 5.0 rad during merger/ringdown
- Amplitude uncertainty 10%.
- Eccentricity ≈ 0.008 (five times other MP codes)
- This waveform provides a lower bound on currently acceptable NR errors.

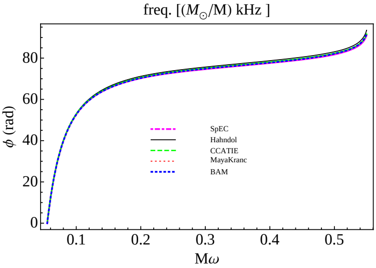
Direct comparison of numerical phase and amplitude

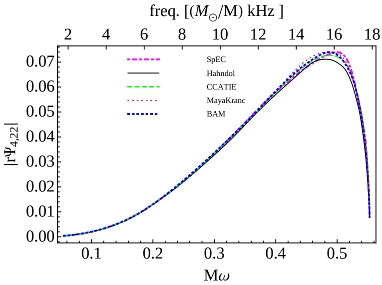
Decompose the $\ell = 2, m = 2$ mode of Ψ_4 as

$$r\Psi_4 = A(\omega)e^{-i\phi(\omega)},$$

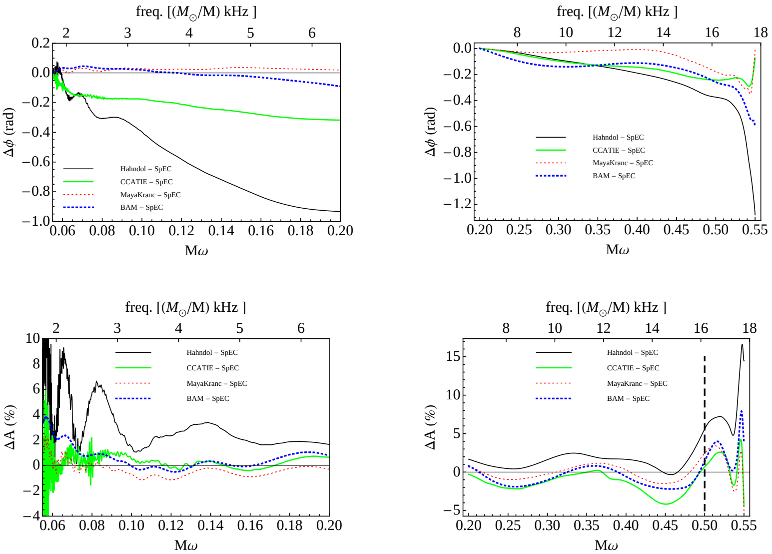
where $M\omega$ is the GW frequency
(roughly twice the orbital frequency during inspiral).

The parametrization in terms of $M\omega$ removes much of the ambiguity in aligning two waveforms for comparison.





Phase and amplitude comparison



Direct comparison: conclusions

- Amplitude and phase disagreements are all within the stated uncertainty estimates
- The errors are larger during merger/ringdown than during inspiral
- The errors are roughly consistent with expectations for each waveform
- The largest total phase disagreement is 1.36 radians.
- The largest amplitude disagreement is about 7%.

Now let's see what this means for GW detection and parameter estimation...

For detection: the match calculation

If we have the Fourier transforms of two waveforms, $x(f)$ and $y(f)$,
and the noise power spectral density of a detector, $S_n(f)$,
we can define the following inner product:

$$\langle x|y\rangle := 4\,\mathrm{Re}\left[\int_{f_{\min}}^{f_{\max}} \frac{\tilde{x}(f)\tilde{y}^{\star}(f)}{S_n(f)}\,df\right].$$

We then define the best match as

$$\mathcal{M} = \max_{\tau,\Phi} \frac{\langle x|y\rangle}{\sqrt{\langle x|x\rangle\langle y|y\rangle}},$$

where we maximize over time and phase offsets.
i.e., we line up the two waveforms to get the best agreement between them.

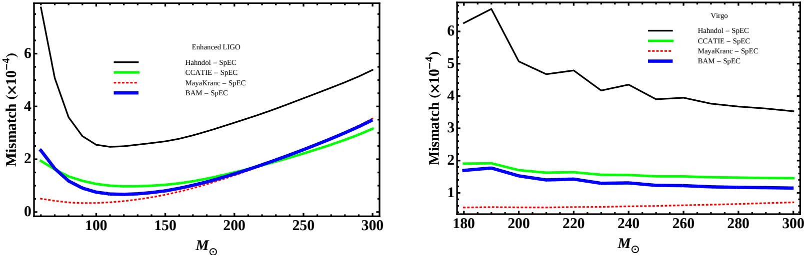
- If they are identical, then $\mathcal{M}(x,y) = 1$.
- Assume that x is a physical waveform in the detector data,
- and that we search for it by matched filtering against a model waveform y
- then if we know that $\mathcal{M} > 0.97$, then we will find the signal 90% of the time.

The match between the SpEC waveform and the others

We look at the mismatch $(1 - \mathcal{M})$.

For current template-bank construction, we want mismatches $< 5 \times 10^{-3}$.

[Lindblom et al, PRD 78, 124020 (2008)]



Conclusion:

Current waveforms *easily* meet the accuracy requirements for detection!

Accuracy requirements for parameter estimation

You've detected GWs from a black-hole binary. Congratulations!

Now you want to estimate the black holes' masses and spins,
and the binary's location and orientation in the sky.

How accurately can we estimate these parameters with our waveforms?

- If for two waveforms $\delta h(t) = h_1(t) - h_2(t)$ satisfies $\langle \delta h | \delta h \rangle < 1$, then they are indistinguishable. [Lindblom et al, PRD 78, 124020 (2008)]
- This is a measure of the accuracy for parameter estimation.

Warning:

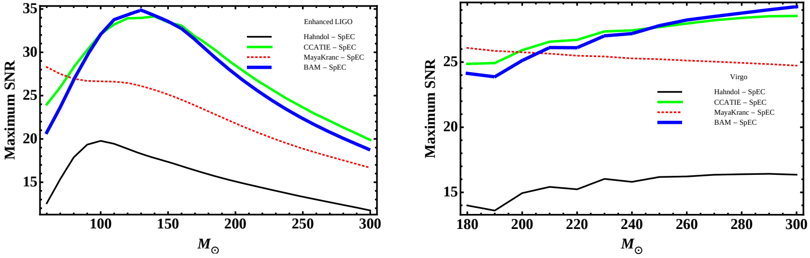
- We have to choose a time and phase alignment when constructing $\delta h(t)$.
- If we align the waveforms to minimize $\langle \delta h | \delta h \rangle$,
- then we can only make statements about estimation of the *intrinsic* parameters of the binary (i.e., the masses and spins).

Distinguishability of the Samurai waveforms

The distinguishability criterion $\langle \delta h | \delta h \rangle$ scales with the source's distance.

So does the signal-to-noise ratio (SNR).

We can calculate the SNR above which these signals could be distinguished...



- SNRs above 10 will be a pleasant surprise with Initial and Enhanced LIGO
- Even with Advanced LIGO, SNRs above 30 will be rare.
- So: these waveforms are unlikely to be distinguishable in detector searches for at least the next five years.

Conclusions

- The numerical waveforms we studied agreed within their stated uncertainties
- They are well within the standard accuracy requirements for detection
- Unless we detect very strong signals, they will be indistinguishable for parameter estimation for at least the next five years.
- By that time, NR waveforms may be orders of magnitude more accurate

But...

- We have not considered the accuracy of higher modes (which also affect parameter estimation)
- These waveforms are for only one BBH configuration
- Much longer waveforms are needed for searches of binaries with $M < 70 M_{\odot}$.
- Those require input from post-Newtonian theory...
- ...for which uncertainty estimates are higher and less robust.

The main conclusion is: the *accuracy* of current numerical simulations is not the bottleneck in producing adequate waveforms for GW searches.