Understanding the "anti-kick" in the merger of binary black holes

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Kick X Anti-Kick

- **Kick**: recoil of the BH due to anisotropic emission of GW
- **Anti-Kick**: deceleration before reaching the final speed (usually after a single BH is formed)
- Many works just give a catalog of the recoil without a physical explanation for the anti-kick
The Basic Idea

(Conjecture)

• At coalescence a single deformed BH is formed, i.e. a BH with an anisotropic distribution of geometric quantities (Horizon Curvatures, $\Psi_2$)

• Asymptotically all deformation must be dissipated (via emission of GW) to leave a Kerr (or Schwarzschild) BH

• The emission of the GW will reflect the anisotropic distribution of the deformed BH’s curvature, which will therefore dictate the directionality
(1) Small BH moves faster, thus it is more efficient on beaming the radiation (Wiseman ‘92). Linear momentum is radiated mostly downwards, therefore leading to an upwards recoil of the system. (collision on the z-axis)
Head on collision
(unequal masses and non spinning black holes)

(2) At the merger the curvature is higher in the upper hemisphere of the distorted BH and linear momentum is radiated mostly upwards leading to the anti-kick

(3) The BH decelerates till a uniform curvature is restored on the horizon
Robinson Trautman Spacetime

The RT space-time is the class of solutions of the vacuum Einstein equations admitting a congruence of null geodesics which are hypersurface orthogonal, shear-free but with expansion

\[ ds^2 = - \left[ K(u, \Omega) - 2r \frac{Q(u, \Omega)_u}{Q(u, \Omega)} - \frac{2M_\infty}{r} \right] du^2 - 2 du dr + r^2 Q(u, \Omega)^{-2} d\Omega^2 \]

\[ K(u, \Omega) = Q^2 \left( 1 + \nabla^2_\Omega \ln Q^2 \right) \]

\[ \partial_u Q = - \frac{Q^3 \nabla^2_\Omega K}{12M_\infty} \]

- Given \( Q(u_0, \Omega) \) evolve for \( u \geq u_0 \)
- Assume axisymmetry
- \( x = \cos \theta \)
Robinson Trautman Spacetime

- Radiative Space-Time (outgoing waves)

\[ R_{abcd} = \frac{1}{r} N_{abcd} + \frac{1}{r^2} III_{abcd} + \frac{1}{r^3} D_{abcd} \]

- Dynamical Horizon: world-tube of past apparent horizon (AH)

\[ Q^2 \nabla^2 \ln R = K - \frac{2M_{\infty}}{R} \]

- Bondi-Momentum

\[ P^\alpha(u) = \frac{M_0}{4\pi} \oint \frac{\eta^\alpha}{Q^3} d\Omega \]

\[ \eta^\alpha = [1, \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta] \]

\[ v^i = \frac{P^i}{P^0} \]
Results

Using ID “reminiscent” of a head-on collision of BHs, we looked at the evolution the horizon curvature and of the recoil.

\[ ds^2|_{AH} = \left[ \frac{R(u, \theta)}{Q(u, \theta)} \right]^2 d\Omega^2 \]

\[ K_{AH} = \frac{2M_\infty}{Q(u, \theta)^3} \]

The intrinsic curvature is different at the N-S poles and is radiated exponentially fast. When the curvature is uniform across the horizon, the recoil reaches its final value.
Results

The recoil $v_k$ as a function of the mass ratio $\nu$ has a well-known behavior:

$$v_k = A\nu^2 \sqrt{1 - 4\nu(1 + B\nu)}$$

It is puzzling that two different values of $\nu$ lead to the same $v_k$ and indeed the curvature on the horizon varies with $\nu$.

It's possible to see two branches: small (large) curvature and large (small) gradients.
Mass Multipoles

• Deformation on the apparent horizon: quantities that does not depend on the coordinates chosen on the 2-surface (S)

\[ ds^2 \bigg|_S = q_{AB} dx^A dx^B \quad q = \text{det} \left| q_{AB} \right| \]

• Axisymmetry provides invariantly defined coordinates \((\tilde{\theta}, \tilde{\phi})\) (where spherical harmonics have the usual orthogonal properties)

\[ \partial_x \tilde{x} = \frac{\sqrt{q}}{R_{AH}^2} \quad \tilde{x}(1) = 1 \quad x = \cos \theta \quad \tilde{x} = \cos \tilde{\theta} \]

• Mass Multipoles: decomposition of the Ricci scalar (intrinsic curvature) on the Legendre polynomials

\[ M_\ell \propto \int_S P_\ell(\tilde{\theta})^2 R(\tilde{\theta}) \]
Results

The mass ratio does not seem to be a good parameter. There should be some physical quantity with the same property on the two branches.

With the mass modes we construct the “effective curvature parameter”. It is injective with the recoil and the maximum velocity is produced by the maximum of the effective curvature.

\[ K_{\text{eff}} = f(M_{\text{even}})g(M_{\text{odd}}) \]

f and g linear on the mass modes \( \Rightarrow \) “deformation” x “gradient”
Results

But....RT spacetimes are surely special (white hole, no angular momentum, no knowledge for $u < u_0$ ...) and clearly not suitable to describe binary BHs. However a comparison can be made! The agreement with the anti-kick given by the close-limit approximation (Le Tiec et al 2010) is not perfect...

Though, it is remarkable the similarities given that the two curves are only related from a logical point of view
Future work

• Understand the dissipation of the curvature

• Study simulations of head on collision in full GR

• Add spins and orbital angular momentum $\Rightarrow$ mass currents given by $\mathcal{G}(\Psi_2)$