Falloff of the Weyl Scalars in Binary Black Hole Spacetimes

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Overview

- Peeling theorem, under general conditions, gives falloff rates of Weyl scalars Ψ_0 - Ψ_4 with radius
- Only $\Psi_4 \sim O(r^{-1})$ nonzero flux at $\mathscr{I}^+$ $\Rightarrow$ outgoing gravitational radiation
- Numerical binary black hole solutions:
  - Spacetimes satisfy general conditions of theorem?
  - Dynamical coordinates suitable for applying the theorem?
- In this work:
  - Compute scalars for numerical BBH spacetime
  - Measure falloff with radius. See also Pollney et al. Phys.Rev. D80 (2009) 121502 arXiv:0910.3656
- Falloff rates consistent with theorem: suggests spacetime and coordinates compatible with assumptions
- Standard NR Ψ_4 method for gravitational radiation is supported

The Weyl Scalars

- 5 complex scalars measuring curvature of the spacetime
- Contractions of Weyl tensor C_{abcd} with a complex null tetrad (l^a, n^a, m^a,$\bar{m}^a$)

Ψ_0	$C_{abcd}\, l^a\, m^b\, l^c\, m^d$	Incoming transverse radiation
Ψ_1	$C_{abcd}\, l^a\, n^b\, l^c\, m^d$	Incoming longitudinal radiation
Ψ_2	$C_{abcd}\, l^a\, m^b\, \bar{m}^c\, n^d$	"Coulomb" part of field
Ψ_3	$C_{abcd}\, l^a\, n^b\, \bar{m}^c\, n^d$	Outgoing longitudinal radiation
Ψ_4	$C_{abcd}\, n^a\, \bar{m}^b\, n^c\, \bar{m}^d$	Outgoing transverse radiation

- Null tetrad components:

l^a	Outgoing radial
n^a	Incoming radial
m^a, $\bar{m}^a$	Complex "angular"

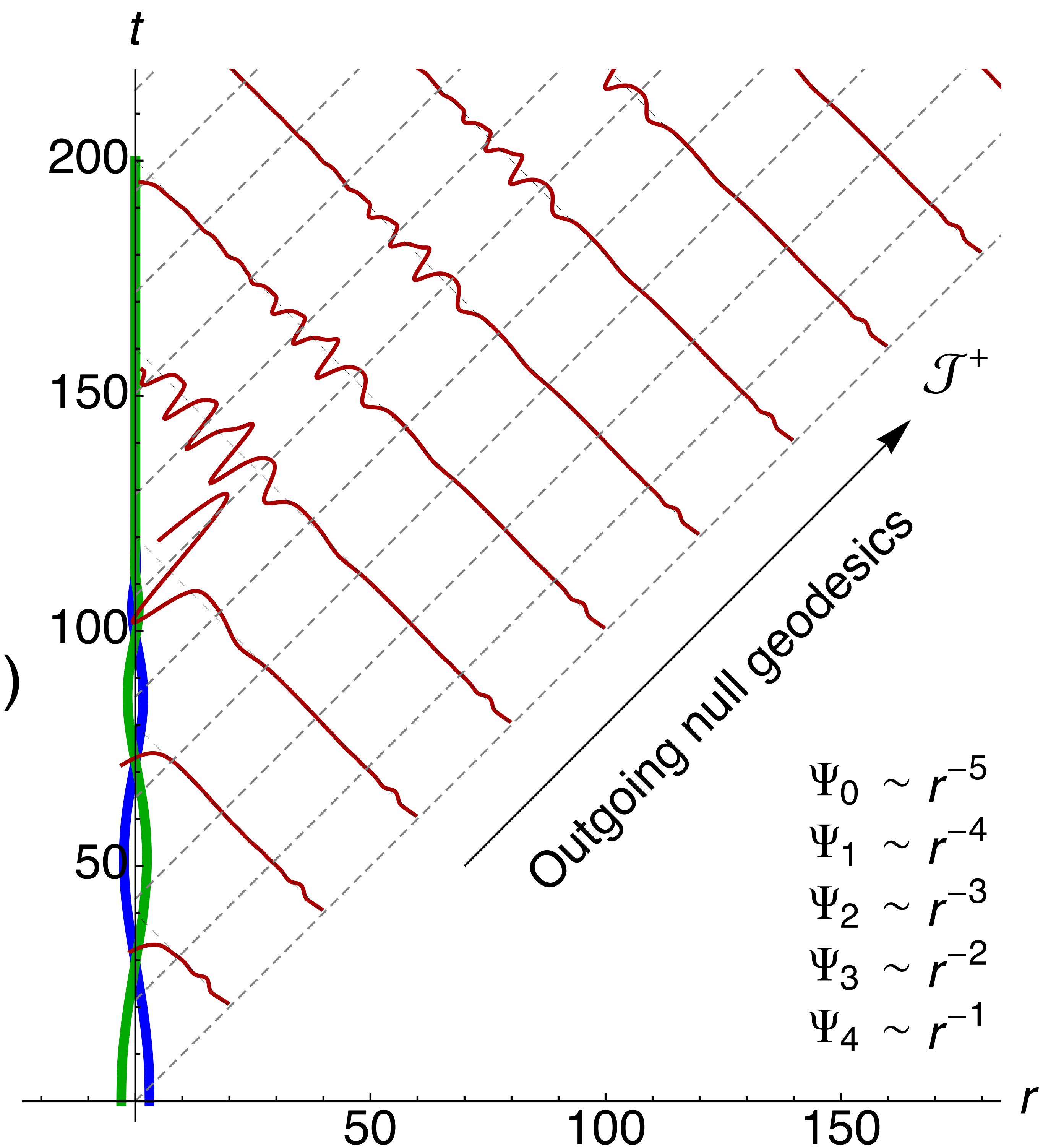
The Peeling Theorem

- Result (Sachs, Newman and Penrose):

$$\Psi_n \sim r^{n-5}$$

near $\mathscr{I}^+$ on outgoing radial null geodesics

- Conditions at $\mathscr{I}^+$:
  - $\bar{C}_{abcd}$ falls off at least as $O(r^{-1})$
  - r defined from conformal transformation at $\mathscr{I}^+$
  - Tetrad n^a, l^a aligned with ingoing/outgoing radial null directions



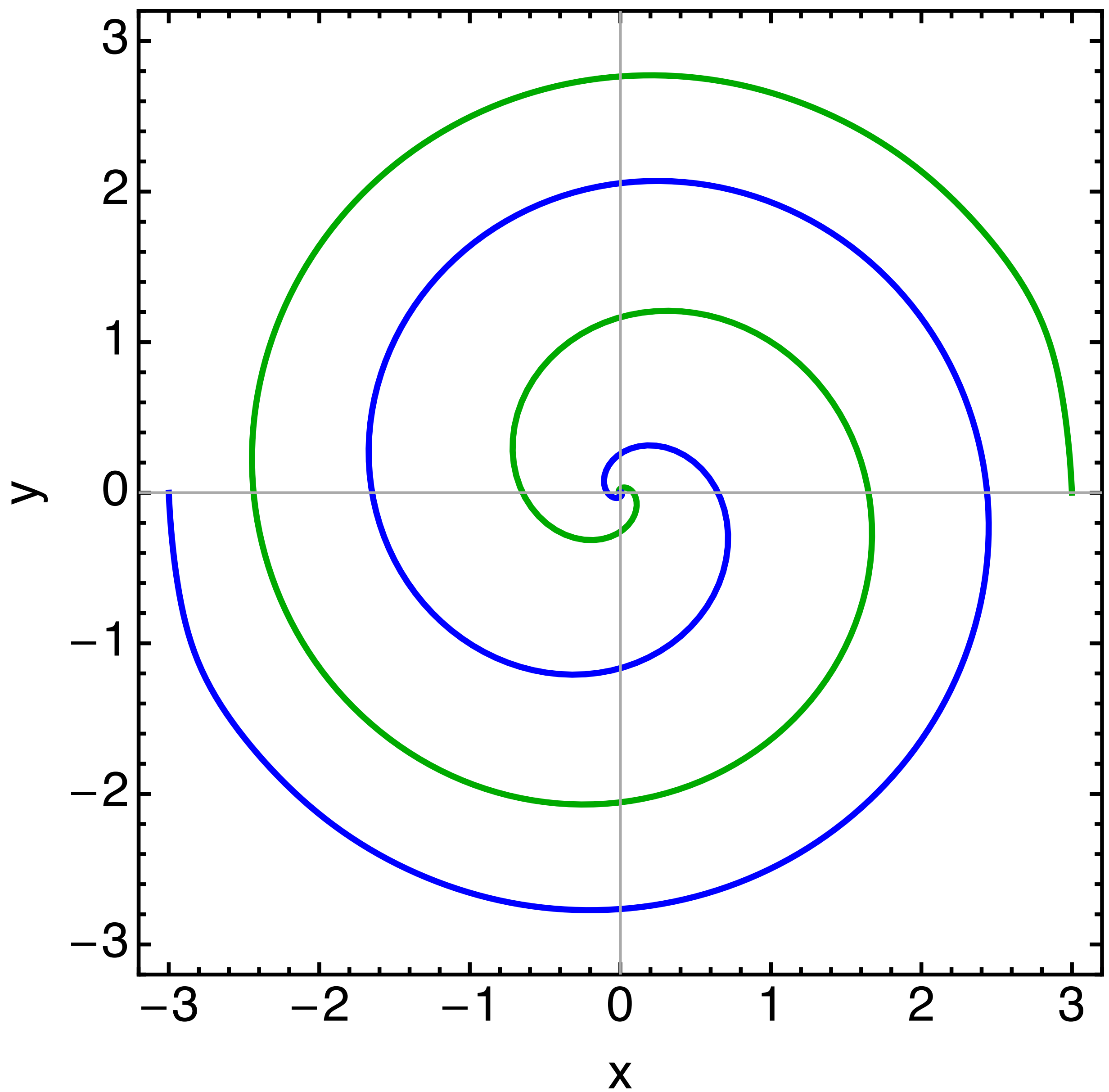
Weyl Scalars in Numerical Relativity

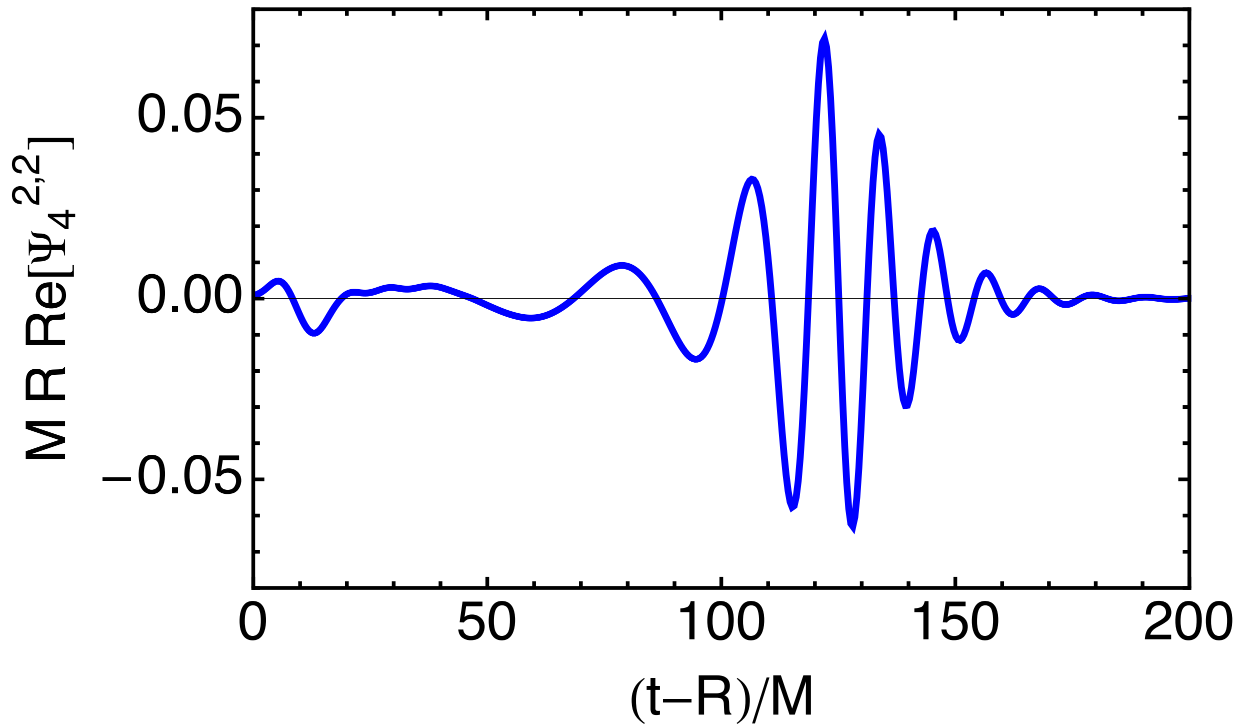
- Tetrad usually used in NR:
  - Construct spatial triad from dynamical coordinate basis vectors (e_r, e_φ, e_θ) and orthonormalise using 3-metric γ_{ab}
  - Form null tetrad from spatial triad and unit hypersurface normal
- Issues:
  - How does this tetrad behave on approach to $\mathscr{I}^+$?
  - Is the radial coordinate r compatible with the theorem?
- Can we use the usual techniques of NR to demonstrate the peeling theorem?

Computing the Scalars

- WeylScal4 code:
  - Computes Ψ_0 - Ψ_4 during simulation in Cactus framework
  - Written by Tanja Bode and IH at Penn State
  - Kranc automated code generation (kranccode.org)
  - Freely available under GPL in the Einstein Toolkit (einsteintoolkit.org)
- For this project:
  - Enabled and tested Kranc multi-patch support in WeylScal4
  - Verified WeylScal4 using Psikadelia and analytic result for I-invariant in Kerr

Black Hole Binary

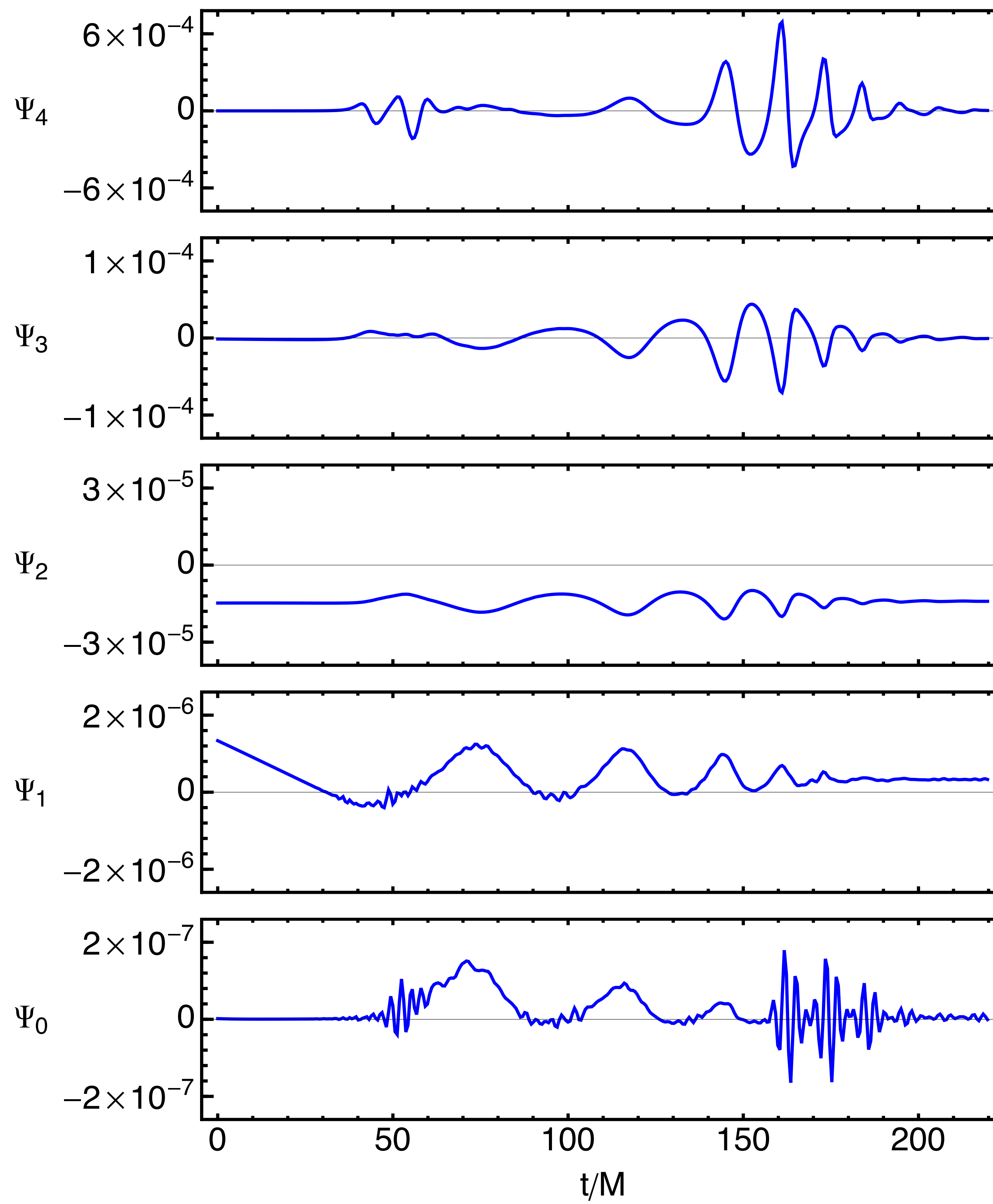




- Simple test case
- Equal-mass, non-spinning
- ~ 1-2 orbits

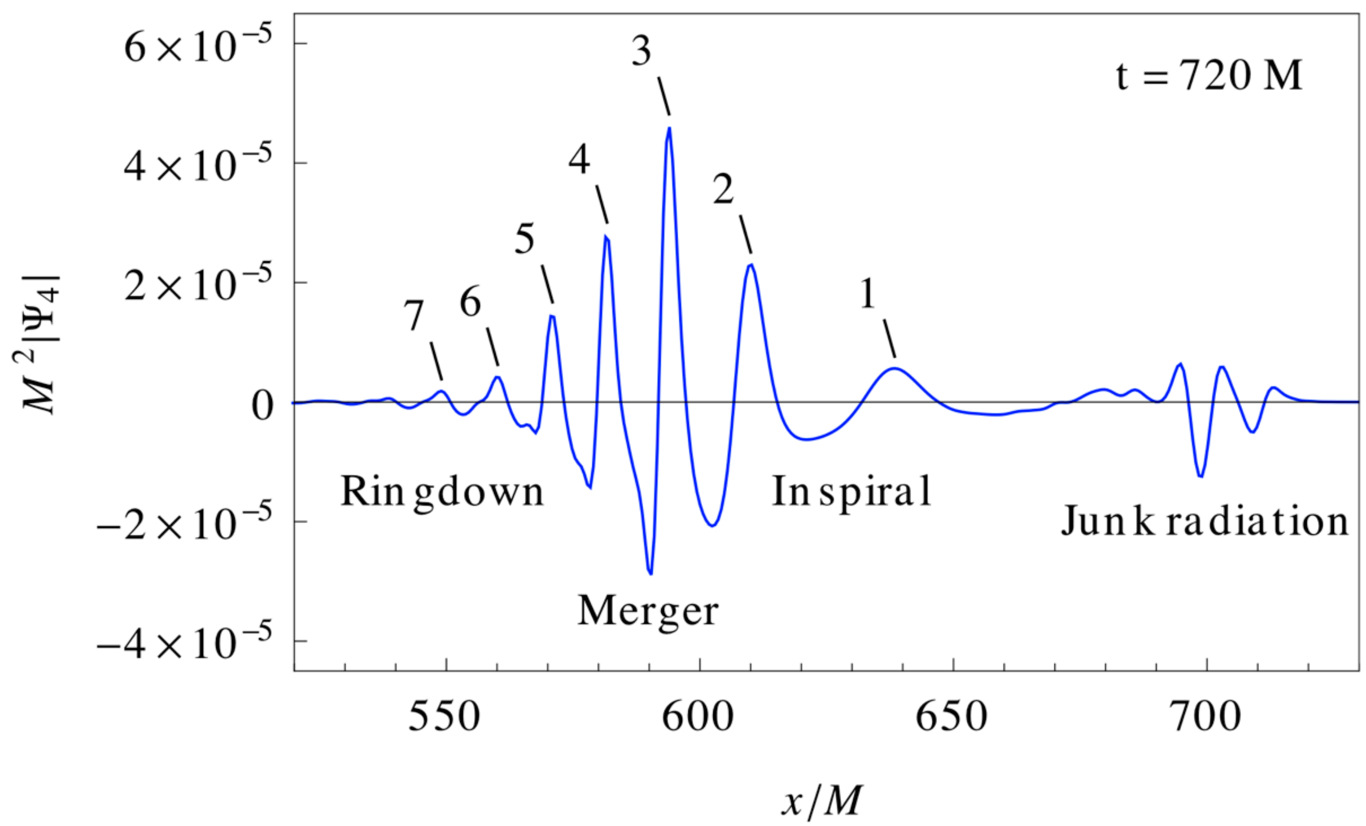
Comparison of the Scalars

- $\Psi_n(t)$ at r = 40 M
- How do the scalars fall off with radius along outgoing null directions?



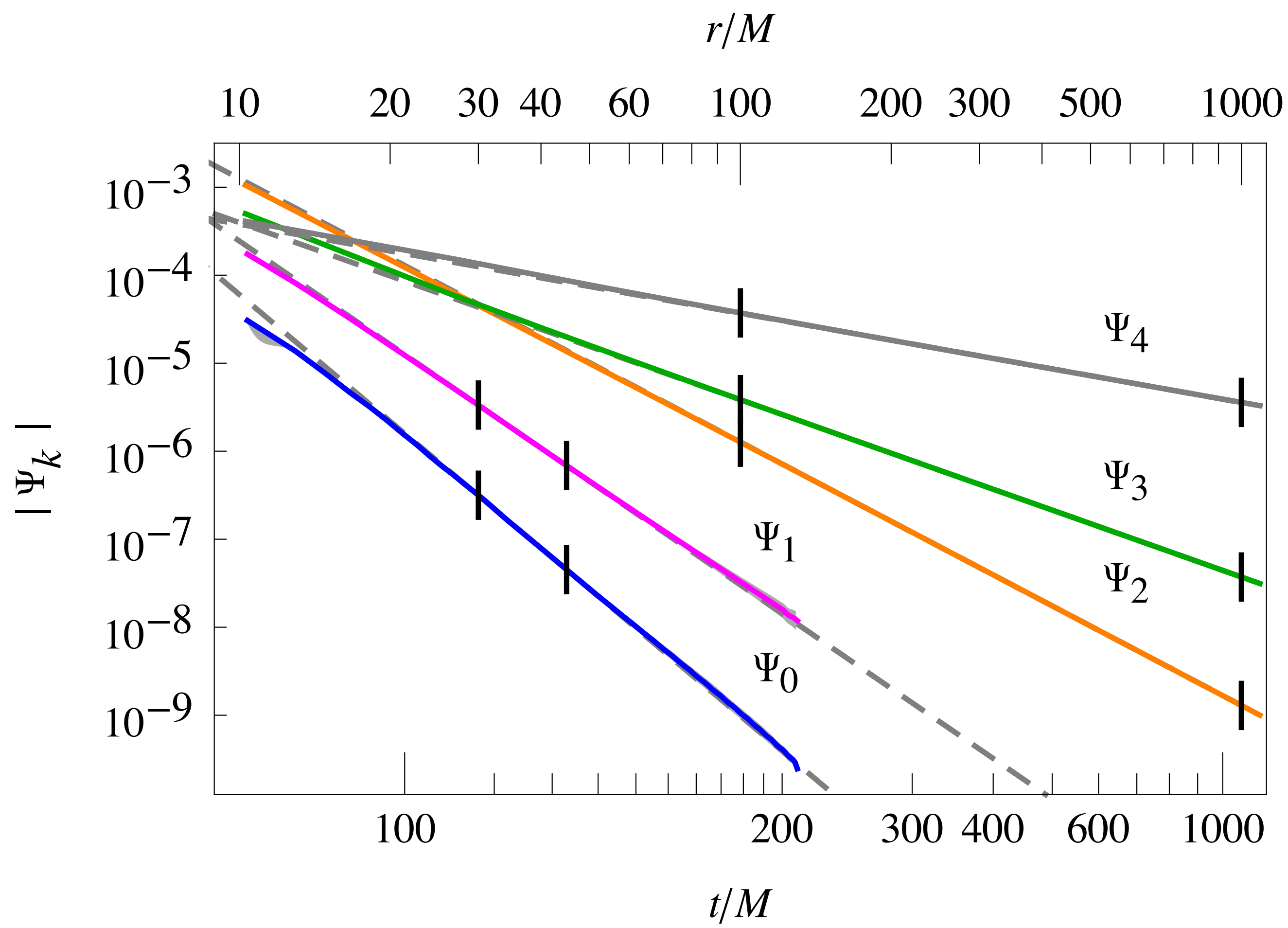
Approximate Null Geodesics

- Peeling theorem, $\Psi_n \sim r^{n-5}$, applies on outgoing radial null geodesics
- We do not calculate geodesics numerically
- Coordinate approximations unsuccessful for r < 100 M
  - t = r + const
  - t = r*(r) + const (Schwarzschild/Kerr tortoise coordinate)



- Successful approximation:
  - Assume that Ψ_4 propagates on null geodesics
  - Define approximate null geodesic $\lambda_i : t \rightarrow r$ as path traced by peak i of Ψ_4 on x-axis

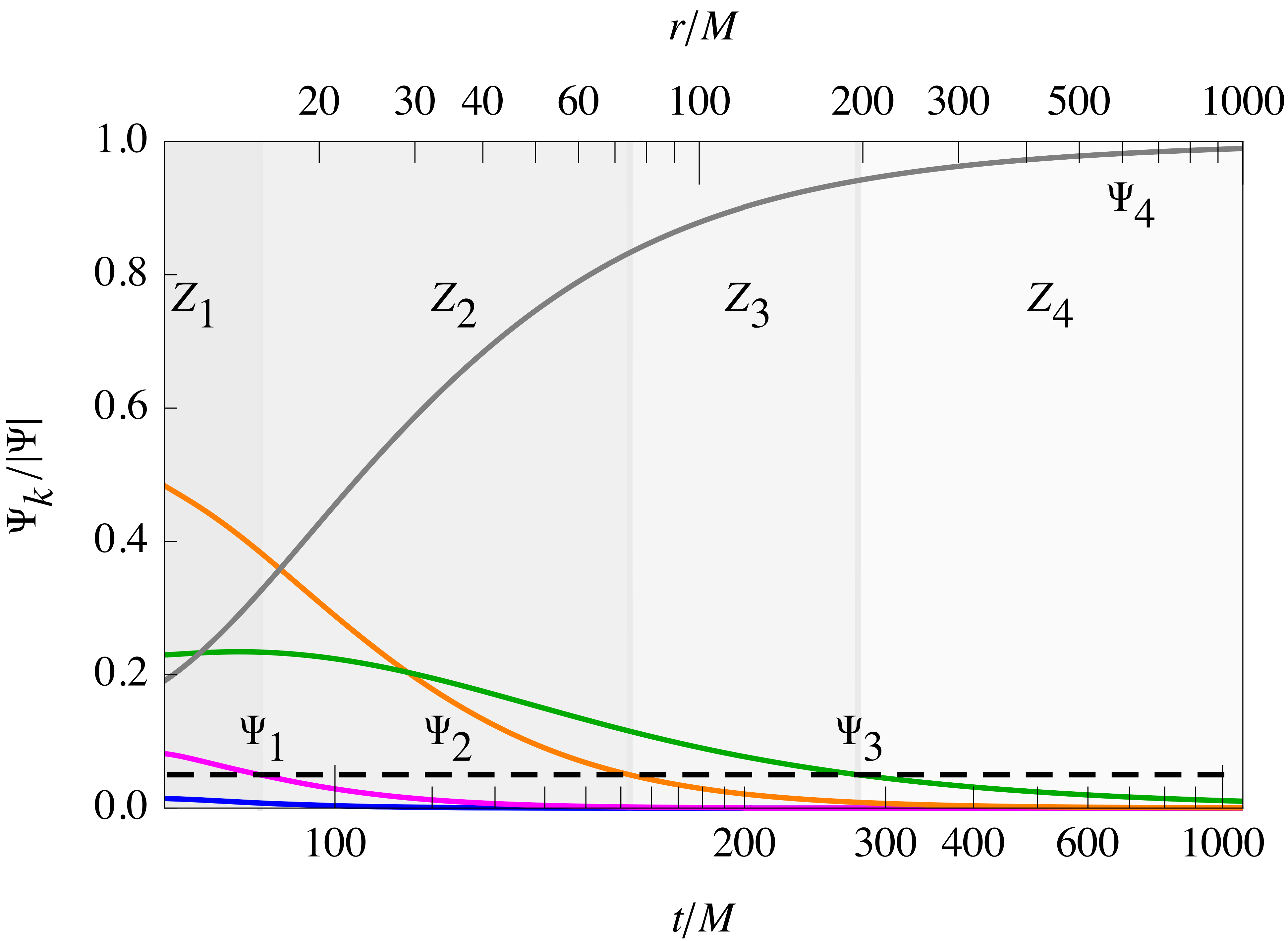
Falloff Rates



Ψ	Falloff rate p	
	Expected	Measured
0	5	4.8(4)
1	4	3.91(7)
2	3	2.99307(6)
3	2	2.0135(6)
4	1	1.01333(7)

- Approximate null geodesic λ_1
- $\Psi_n \sim r^{-p}$, theorem: $p = 5-n$
- Predicted falloff rates, $\Psi_n \sim r^{n-5}$, obtained within 4%
- Discrepancies due to finite-radius fitting

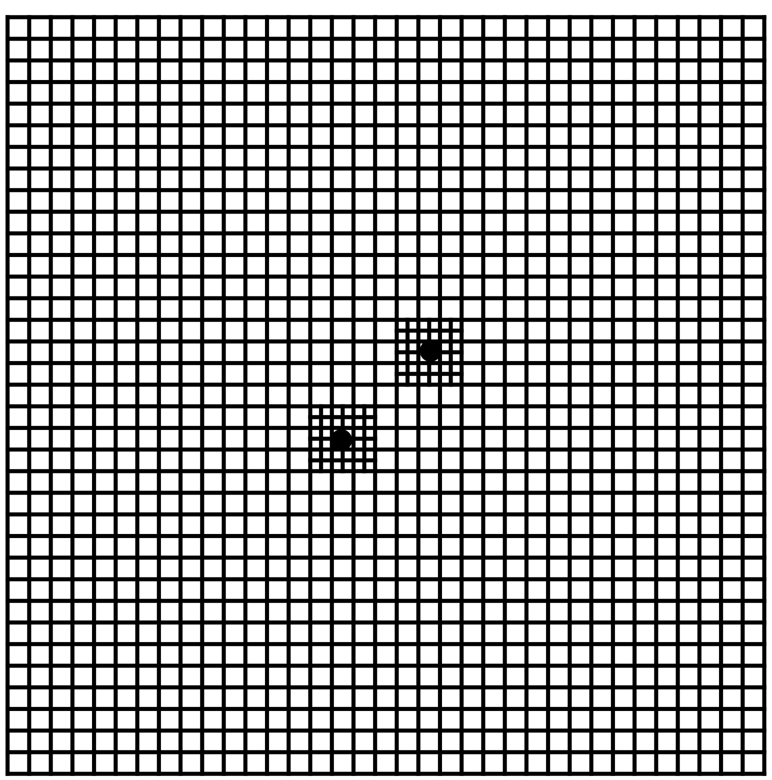
Curvature Zones



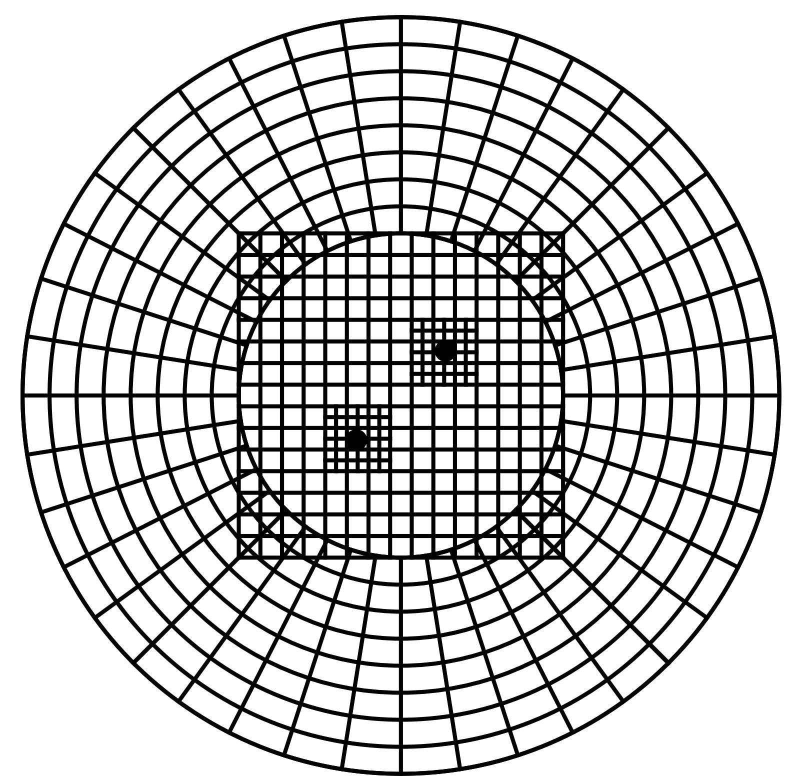
- Z_n is zone where Ψ_n contributes > 5% to $\Sigma |\Psi_k|$

Technical Challenges

- Ψ_0 and Ψ_1 fall off very fast (r^{-5} and r^{-4}) - very low amplitude by r ~ 70 M
- Contaminated by numerical errors from inter-patch boundary
- Ψ_4 falls off as r^{-1} - need large radius to see correct rate
- Single Cartesian patch simulation for Ψ_0 and Ψ_1 with tapered grids
- Multi-patch simulation for Ψ_2 - Ψ_4



(not to scale)



Summary

- Have computed the Weyl scalars Ψ_0 to Ψ_4 for a binary black hole spacetime
- Required careful numerical treatment for Ψ_0 and Ψ_1
- Measured falloff along approximate null geodesics corresponding to peaks in Ψ_4
- Obtained rates consistent with the peeling theorem
- Identified qualitative curvature zones
- Confidence in standard NR techniques and interpretation of Ψ_4 from simulations as gravitational waves